Final Exam of Data Structures (CSE, NTOU)

Student ID: _____

09:20 – 12:05, 25 December 2024; Room INS105

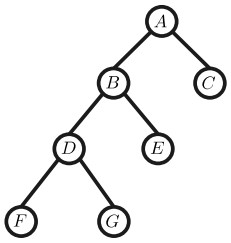
Name: _____

Note: Cell phones and any calculator are forbidden.

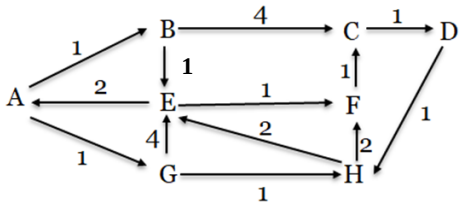
1. (5%) Please compute the number of different ways to compute the product of 8 matrices.

2. (10%) Let $f(n) = \binom{n}{\alpha n}$ for $\alpha \in (0,1)$, say $f(n) = O(\lambda^n \cdot g(n))$ for some polynomial $g(n)$.
   (a). Please give the exact formula for the value of λ.
   (b). Please compute the value of λ for $\alpha = 0.5$.
   Hint: Using the Stirling's approximation that $\sqrt{2\pi n}(n/e)^n < n! < \sqrt{2\pi n}(n/e)^n e^c$ for some $c > 0$.

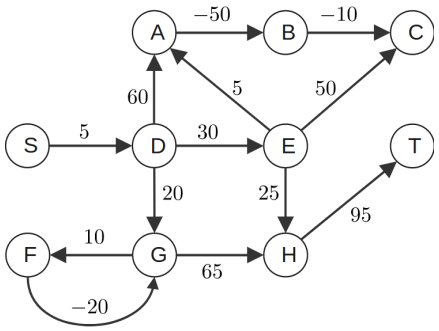
3. (15%) Give the **inorder**, **preorder**, **postorder**, and **level order** travels for the following binary tree.



4. (10%) A weighted directed graph G is given as follows. Please apply Dijkstra's algorithm to find **shortest paths** from ***A*** to ***C*** and from ***A*** to ***H***. If more than 1 vertices have the same minimum distance, choose the vertex with alphabet priority. (**Note**: You should give two "**paths**", each of them is represented as a sequence of vertices.)



5. (15%) Given the directed graph as below, please use Bellman-Ford algorithm to compute the **distances** from node *S* to node ***A***, node ***C*** and node ***T*** (*You should give or explain the explicit steps of the algorithm*).



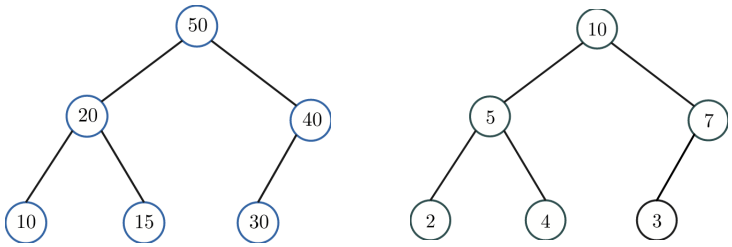
6. (10%) Let x and y are the **most far-away** nodes in a **binary tree** T. Let $d_T(x,y)$ denote the length of the shortest path from x to y in T. What is the **maximum possible** value of $d_T(x,y)$? What is the **minimum possible** value of $d_T(x,y)$? Justify your answers rigorously.

7. (10%) Prove that any binary tree of n nodes can be **uniquely determined** by its inorder traversal and preorder travel sequences. (*Hint*: *Mathematical Induction on n*).

8. (5%) Explain that **why** we adopt an **array** to implement the **heap** data structure.

9. (10%) Consider the max heaps below.

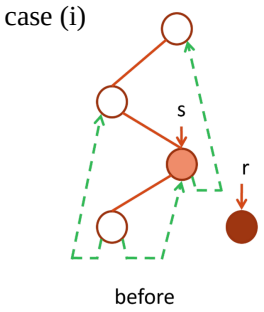
   (a). Please draw the heap after inserting element of key 55 in heap (i).
   (b). Please draw the heap after deleting the top element in heap (ii).

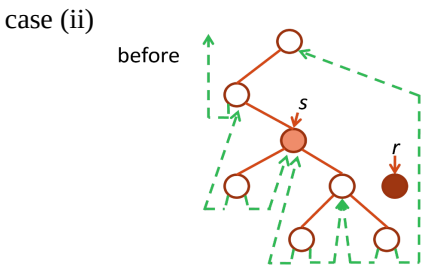


(i) (ii)

10. (10%) Assume that `insucc()` is the function which can identify the inorder successor of a node in a threaded binary tree. Please complete the function `insertRight()` which inserts a node `r` as the right child of a node s in the threaded binary tree. (10 spaces; each for 1%)

```
void insertRight (threadedPointer s, threadedPointer r) {
    ________________ temp;
    r->rightChild = ____________________, r->rightThread = ____________________;
    r->leftChild = ____________________, r->leftThread = ____________________;
    s->rightChild = ____________________, s->rightThread = ____________________;
    if (____________________) {
        temp = _________________
        temp->_________________
    }
}
```





11. (10%) Show that building a max-heap of n numbers can be done in $O(n)$ time:
    (a). Provide your $O(n)$ time algorithm.
    (b). Analyze the complexity of your algorithm. (*Hint*: You might need to prove $\sum_{x=0}^{\infty} x/2^x = O(n)$ as well.)

12. (10%) Consider Prim's algorithm for finding a minimum-cost spanning tree in a weighted undirected graph.
    (a). Please write down Prim's algorithm (pseudo-code is fine).
    (b). Please analyze the time complexity of Prim's algorithm.

13. (5%) Consider the following representation for two disjoint sets. Please draw the result after performing `weightedUnion(0,5)` and `collapsingFind(8)`.