Depth-First Search and Breadth-First Search

(DFS & BFS)

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Outline

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Elementary Graph Operations

Reachability

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- **Goal**: visit all vertices in G that are reachable from v.

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- In the following discussion, we shall assume that the linked adjacency list representation for graphs is used.

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Depth First Search (DFS) (1/2)

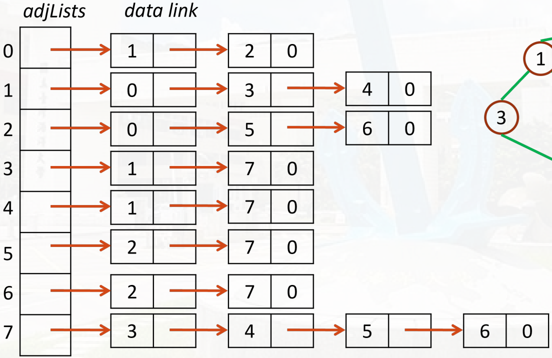
- We begin the search by visiting the start vertex, v.
- Next, we select an unvisited vertex, w, from v's adjacency lists and carry out a DFS on w.
- We preserve our current position in v's adjacency list by placing it on a **stack**.
- Eventually our search reaches a vertex, say u, that has no unvisited vertices on its adjacency list.

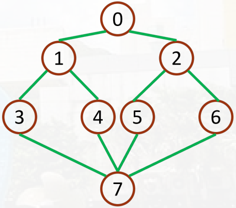
Depth First Search (DFS) (2/2)

- At this point, we remove a vertex from stack and continue processing its adjacency list.
- Previously visited vertices are discarded; unvisited vertices are visited and placed on the stack.
- The search terminates when the stack is empty.

DFS Example

- Using a stack and recursion.
    - It resembles the preoder tree traversal.





- The DFS order: $v_0 \to v_1 \to v_3 \to v_7 \to v_4 \to v_5 \to v_2 \to v_6$.

The Pseudocode of DFS

```
DFS(G, u) {
    u.visited = True
    for each v in G.Adj[u]
        if v.visited == False
            DFS(G, v)
}

driving main () {
    for each u in G
        u.visited = false
    for each u in G
       DFS(G, u)
}
```

DFS in C

```c
#define FALSE 0
#define TRUE 1
short int visited[MAX_VERTICES];
/* intializing to be FALSE for all */

void DFS(int v)  {
/* DFS beginning at vertex v */
    nodePointer w;
    visited[v] = true;
    printf("%5d",v);
    for(w = graph[v]; w; w = w->link)
        if (!visited[w->vertex])
            DFS(w->vertex);
}
```

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  - One needs to scan the corresponding row of the adjacency matrix.
- Total time: $O(n^2)$.

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Breadth First Search (BFS) (1/2)

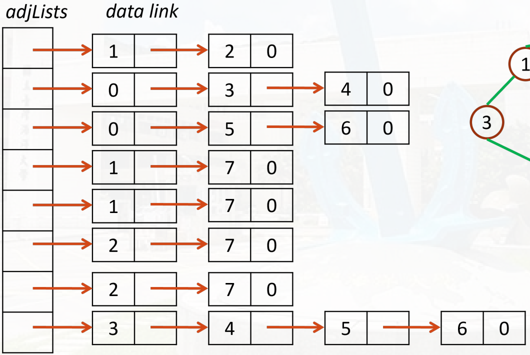
- The algorithm starts at vertex v and marks it as visited.
- Then visiting each of the vertices on v's adjacency list.
- When we have visited all the vertices on v's adjacency list, we visit all the unvisited vertices that are adjacent to the first vertex on v's adjacency list.
- To implement this scheme, as we visit each vertex we place the vertex in a queue.

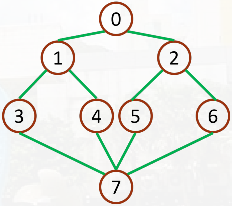
Breadth-First Search (BFS) (2/2)

- When we have exhausted an adjacency list, we remove a vertex from the queue and proceed by examining each of the vertices on its adjacency list.
- Unvisited vertices are visited and placed on the queue; visited are ignored.
- Finish the search when the queue is empty.

BFS Example

- Using a queue.
  - It resembles the level-order tree traversal.





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The Pseudocode of BFS

```
BFS(G, u) { // let Q be the queue
    Q.enqueue(u)
    u.visited = True
    while (Q.empty() == False) { // when Q is not empty
        v = dequeue(Q)
        for all w in N(v) {
            if (w.visited == False) {
                Q.enqueue(w)
                w.visited = True
            }
        }
    }
}
driving main () {
    for each u in G
        u.visited = false
    BFS(G, u)
}
```

BFS in C

```
void bfs(int v) {
    nodePointer w;
    front = rear = NULL;  /* initialize queue */
    printf("%5d",v);
    visited[v] = TRUE;
    addq(v);

    while (front) {
        v = dequeue();
        for (w = graph[v]; w ; w->link)
            if (!visited[w->vertex]) {
                printf("%5d", w->vertex);
                enqueue(w->vertex);
                visited[w->vertex] = TRUE;
            }
    }
}
```

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  - For the adjacency list representation, this loop has a total cost of $d_0 + d_1 + \ldots + d_{n-1} = O(e)$, where $d_i = \text{degree}(v_i)$.
- For the adjacency matrix representation, the while loop takes $O(n)$ time for each vertex visited.
  - Therefore, the total time is $O(n^2)$.

As was true of DFS, all vertices visited, together with all edges incident to them, form a connected component of G.

Discussions