Linked List

Equivalence Relations, Sparse Matrices & Doubly Linked Lists

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Outline

1. Equivalence Relations
2. Sparse Matrices Revisted
3. Doubly Linked Lists

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Equivalence Relation

A relation over a set S is said to be an equivalence relation over S iff it is symmertric, reflexive, and transitive over S.

- reflexive: $x \equiv x$ for each $x \in S$.
- symmetric: for $x, y \in S$, if $x \equiv y$, then $y \equiv x$.
- transitive: for x, y, z, if $x \equiv y$ and $y \equiv z$, then $x \equiv z$.

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Example

Given $0 \equiv 4$, $3 \equiv 1$, $6 \equiv 10$, $8 \equiv 9$, $7 \equiv 4$, $6 \equiv 8$, $3 \equiv 5$, $2 \equiv 11$, $11 \equiv 1$.

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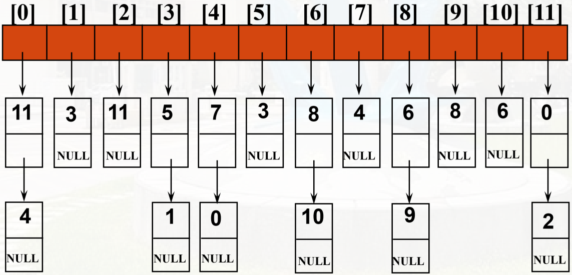
We have three equivalent classes:

$$\{0, 2, 4, 7, 11\}, \{1, 3, 5\}, \{6, 8, 9, 10\}.$$

Lists after Giving Pairs as the Input

$0 \equiv 4, 3 \equiv 1, 6 \equiv 10, 8 \equiv 9, 7 \equiv 4,$
$6 \equiv 8, 3 \equiv 5, 2 \equiv 11, 11 \equiv 0.$

```c
typedef struct node *nodePointer;
typedef struct node {
    int data;
    nodePointer link;
};
```



Outline

Issues for Previous Representation

- When we performed matrix operations such as $+$, $-$, or $*$, the number of **nonzero terms** varied.
- The sequential representation of sparse matrices suffered from the same inadequacies as the similar representation of polynomials.

Solution:

- Linked list representation for sparse matrices.

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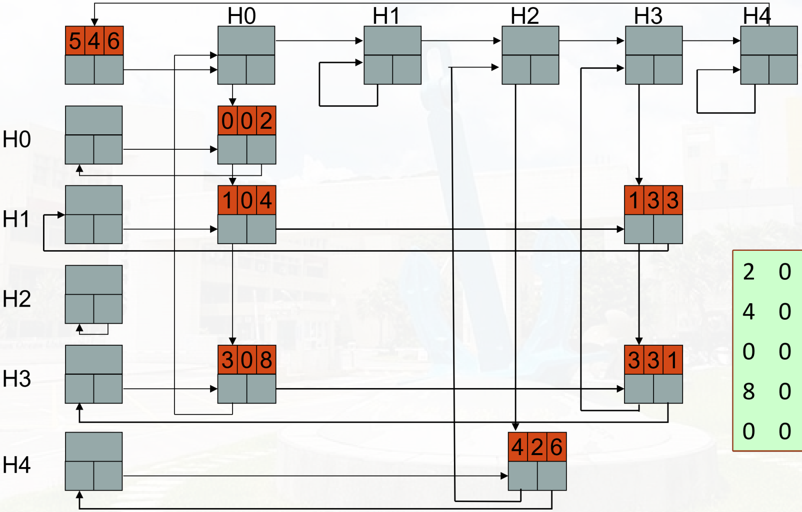
Solution:

- Linked list representation for sparse matrices.
- Two types of nodes in the representation: header nodes and element nodes.

header node　　element node



2	0	0	0
4	0	0	3
0	0	0	0
8	0	0	1
0	0	6	0

Sparse Matrix Representation

- We represent each column (row) of a sparse matrix as a circularly linked list with a header node.
- The header node for row *i* is also the header node for column *i*. The number of header nodes is $\max\{\text{numRows}, \text{numCols}\}$.
- Each element node is simultaneously linked into two lists: a row list, and a column list.
- Each head node is belonged to three lists: a row list, a column list, and a header node list.

Outline

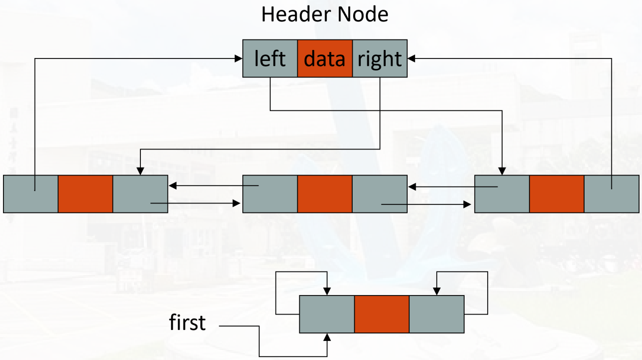
Issues for Singly Linked Lists

- The only way to find the node that precedes some node p is to start at the beginning of the list.
- Sometimes it is necessary to move in either direction.

Doubly linked lists:

```
typedef struct node *nodePointer;
typedef struct node {
    nodePointer llink;
    element data;
    nodePointer rlink;
};
```

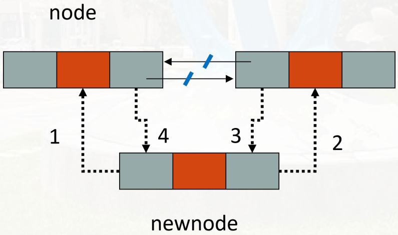
```
ptr = ptr->llink->rlink = ptr->rlink->llink
```



Empty doubly linked circular list with header node

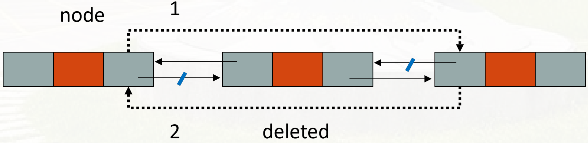
Insertion into a doubly linked circular List

```
void d_LCL_insert(nodePointer node, nodePointer newnode) {
/* insert newnode to the right of node */
    newnode->llink = node;            // 1
    newnode->rlink = node->rlink;     // 2
    node->rlink->llink = newnode;     // 3
    node->rlink = newnode;            // 4
}
```



Insertion into a doubly linked circular List

```
void d_LCL_delete(nodePointer node, nodePointer deleted) {
/* delete from the doubly linked list */
    if (node == deleted)
        printf("Deletion of header node not permitted.\n");
    else {
        deleted->llink->rlink = deleted->rlink;^^I^^I// 1
        deleted->rlink->llink = deleted->llink;^^I^^I// 2
        free(deleted);
    }
}
```



Discussions