

Tree Traversals

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Outline

- 1 Binary Tree Traversals (Using a Stack)
- 2 Level-Order Traversal
- 3 Additional Binary Tree Operations



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Binary Tree Traversals

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 - Six possible combinations: LVR , LRV , VLR , VRL , RVL , RLV .



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How to visit each node of a tree exactly once?

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 - Six possible combinations: LVR , LRV , VLR , VRL , RVL , RLV .
- Adopting the convention that we traverse left before right, only three combinations of VLR remains:



Binary Tree Traversals

Question

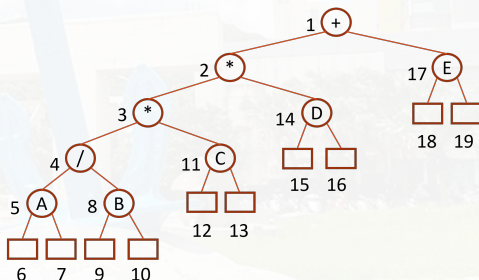
How to visit each node of a tree exactly once?

- Let V , L , R stand for visiting the node, moving left, and moving right, resp.
 - Six possible combinations: LVR , LRV , VLR , VRL , RVL , RLV .
- Adopting the convention that we traverse left before right, only three combinations of VLR remains:
 - **inorder** (中序走訪法)
 - **postorder** (後序走訪法)
 - **preorder** (先序走訪法)



Tree Traversals

- Inorder traversal (*LVR*):
 $A/B * C * D + E$
- Preorder traversal (*VLR*):
 $+ ** / ABCDE$
- Postorder traversal (*LRV*):
 $AB / C * D * E +$

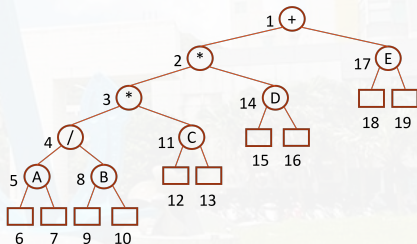


Code for Inorder Traversal

```

void inorder (treePointer ptr) {
/* inorder tree traversal */
  if (ptr) {
    inorder (ptr->leftChild);
    printf ("%d", ptr->data);
    inorder (ptr->rightChild);
  }
}

```



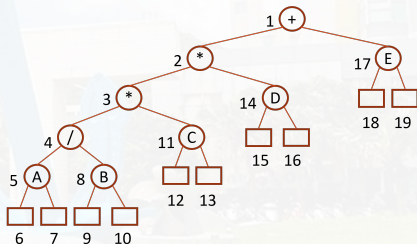
$$A/B * C * D + E$$

Code for Preorder Traversal

```

void Preorder (treePointer ptr) {
/* inorder tree traversal */
  if (ptr) {
    printf ("%d", ptr->data);
    inorder (ptr->leftChild);
    inorder (ptr->rightChild);
  }
}

```



+**/ABCDE

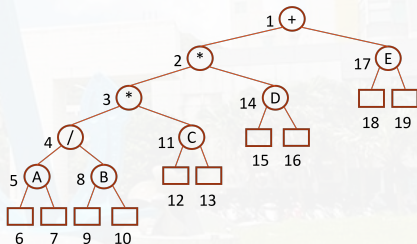


Code for Postorder Traversal

```

void Postorder (treePointer ptr) {
/* inorder tree traversal */
  if (ptr) {
    inorder (ptr->leftChild);
    inorder (ptr->rightChild);
    printf ("%d", ptr->data);
  }
}

```



$$AB/C * D * E +$$

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Level-Order Traversal

- When written recursively, the inorder, preorder, and postorder traversals all require a **stack**.



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- We now turn to a traversal that requires a **queue**. This traversal called **level-order traversal**.



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Steps of a Level-Order Traversal

- 1 visit the root first.
- 2 then the root's left child followed by the right child.
- 3 visit next level from leftmost node to right most node.

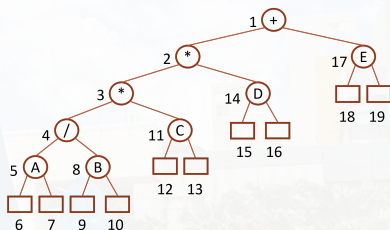


Code for the Level-Order Traversal

```

void levelOrder(treePointer ptr) {
    int front = rear = 0;
    treePointer queue[MAX_QUEUE_SIZE];
    if (!ptr) return; /* empty tree */
    add(ptr); // enqueue
    while (1) {
        ptr = delete(); // dequeue
        if (ptr) {
            printf("%d", ptr->data);
            if (ptr->leftChild)
                // leftChild exists
                add(ptr->leftChild);
                // enqueue
            if (ptr->rightChild)
                // rightChild exists
                add(ptr->rightChild);
                // enqueue
        } else break;
    }
}

```



+

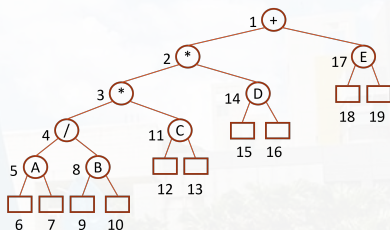


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                // enqueue
        } else break;
    }
}

```



+*

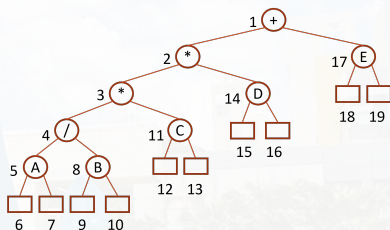


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        } else break;
    }
}

```



+*E

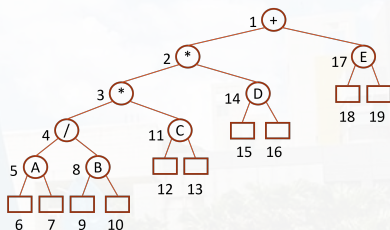


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}

```



++E*

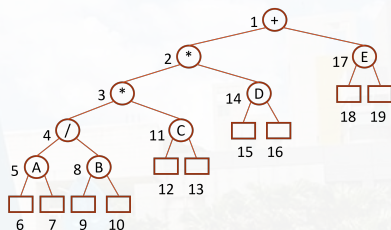


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                add(ptr->rightChild);
                // enqueue
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    }
}

```



++E*D

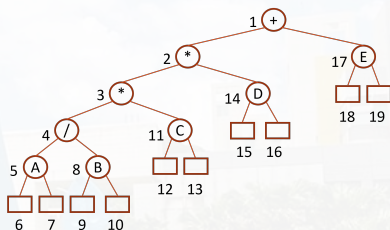


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                add(ptr->rightChild);
                // enqueue
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    }
}

```



$++E * D /$

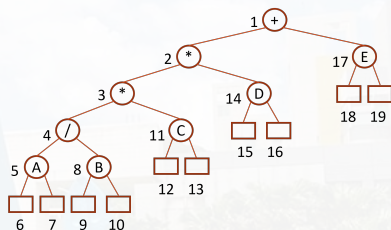


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        ptr = delete(); // dequeue
        if (ptr) {
            printf("%d", ptr->data);
            if (ptr->leftChild)
                // leftChild exists
                add(ptr->leftChild);
                // enqueue
            if (ptr->rightChild)
                // rightChild exists
                add(ptr->rightChild);
                // enqueue
        } else break;
    }
}

```



$++E * D / C$

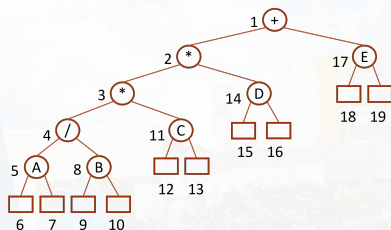


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                // rightChild exists
                add(ptr->rightChild);
                // enqueue
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}

```



$++E * D / CA$

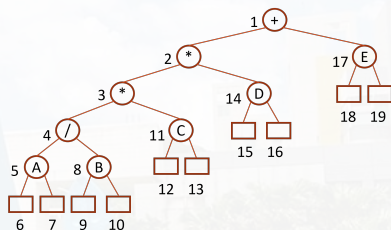


Code for the Level-Order Traversal

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void levelOrder(treePointer ptr) {
    int front = rear = 0;
    treePointer queue[MAX_QUEUE_SIZE];
    if (!ptr) return; /* empty tree */
    add(ptr); // enqueue
    while (1) {
        ptr = delete(); // dequeue
        if (ptr) {
            printf("%d", ptr->data);
            if (ptr->leftChild)
                // leftChild exists
                add(ptr->leftChild);
                // enqueue
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                // rightChild exists
                add(ptr->rightChild);
                // enqueue
        } else break;
    }
}

```



$++E * D / CAB$



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Copying a Binary Tree

```
treePointer copy(treePointer original) {  
    /* return a tree_pointer to an exact copy of the original tree */  
    treePointer temp;  
    if (original) {  
        MALLOC(temp, sizeof(*temp));  
        temp->leftChild = copy(original->leftChild);  
        temp->rightChild = copy(original->rightChild);  
        temp->data = original->data;  
        return temp;  
    }  
    return NULL;  
}
```



Testing for Equality of Binary Trees

```

int equal(treePointer first, treePointer second) {
    /* function returns FALSE if the binary trees first and second are not equal
    Otherwise it returns TRUE */
    return (
        (!first && !second) || (first && second &&
            (first->data == second->data) &&
            equal(first->leftChild, second->leftChild) &&
            equal(first->rightChild, second->rightChild))
    )
}

```

equal(treePointer first, treePointer second)

First->data == second->data

equal(treePointer first->leftChild, treePointer second->leftChild)

equal(treePointer first->rightChild, treePointer second->rightChild)



Discussions

