

Final Exam of Discrete Mathematics

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1. (a) $(2^3)(3^{(3^2-3)/2}) = (2^3)(3^3)$.
(b) $2^{n(n+1)/2}$.
2. (a) $\phi(100) = 40$.
(b) Skip.
3. (a) $5! - \binom{5}{1}4! + \binom{5}{2}3! - \binom{5}{3}2! + \binom{5}{4}1! - \binom{5}{5}0! = 44$.
(b) $n! \left(\sum_{i=0}^n \frac{(-1)^i}{i!} \right)$.
(c) Skip.
4. (a) $\frac{1}{1-x}$.
(b) $\frac{x}{(1-x)^2}$.
(c) $\frac{x^2(x+1)}{(1-x)^3}$.
5. (a) $\binom{-7}{3}(-2)^3 = 672$.
(b) The generating function is $x^4(1-x)^{-4}$, thus the answer is $\binom{-4}{5} = 56$.
6. $a_n = (2 + 5n)3^n, n \geq 0$.
7. $a_n = \left(1 - \frac{1}{8}n + \frac{1}{8}n^2\right) 2^n$.
8. (a) $a_n = a_{n-1} + a_{n-2}, n \geq 2, a_0 = 1, a_1 = 2$.
(b) $a_n = \frac{1}{\sqrt{5}} \left[\left(\frac{1+\sqrt{5}}{2}\right)^{n+2} - \left(\frac{1-\sqrt{5}}{2}\right)^{n+2} \right], n \geq 0$
9. (a) $a_n = 2a_{n-1} + 1, n \geq 1$ and $a_0 = 0$.
(b) $a_n = 2^n - 1, n \geq 0$.