Midterm 1 of Discrete Mathematics

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Note: Please list complete process of the calculation or the proof for each problem.

1. Given an integer n, prove that $\frac{(2n)!}{2^n}$ is an integer. (10%)
2. (a) How many arrangements are there of all the letters in SOCIOLOGICAL? (5%)
   (b) In how many of the arrangements in part (a) are A and G adjacent? (10%)
   (c) In how many of the arrangements in part (a) are all the vowels adjacent? (10%)
3. Evaluate $\sum_{i=0}^{5} \binom{10}{i} \cdot \binom{5}{5-i}$. (10%)
4. Recall that the *Binomial Theorem* states that $(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} \cdot x^k \cdot y^{n-k}$.
   (a) Prove that $\binom{n}{0} + \binom{n}{1} + \ldots + \binom{n}{n} = 2^n$. (5%)
   (b) If n is an even integer, evaluate $2\binom{n}{0} + \binom{n}{1} + 2\binom{n}{2} + \binom{n}{3} + \ldots + 2\binom{n}{n-2} + \binom{n}{n-1} + 2\binom{n}{n}$. (10%)
5. Find the coefficient of $w^2x^2y^2z^2$ in the expansion of $(v + w - 2x + y + 5z + 3)^{10}$. (10%)
6. Determine the number of integer solutions of $x_1 + x_2 + x_3 + x_4 = 25$, where $x_1 > 0$, $x_2 > 2$, $x_3 > 1$ and $x_4 \geq 3$. (10%)
7. Please negate and simplify the statement $\forall x\ \forall y\ [(x > y) \rightarrow (x - y) > 0]$. (10%)
8. For the following statements the universe comprises all nonzero integers. Determine the truth value of each statement (You can just write down TRUE or FALSE in your answer sheet).
   (a) $\exists x\ \exists y\ [xy = 1]$; (5%)
   (b) $\exists x\ \exists y\ [(3x - y = 7) \wedge (2x + 4y = 3)]$. (5%)
9. Consider the primitive statements p, q, r, s, t and the argument

$$\begin{array}{c} p \\ p \vee q \\ q \rightarrow (r \rightarrow s) \\ t \rightarrow r \\ \hline \therefore \neg s \rightarrow \neg t \end{array}$$

   Show that this is an invalid argument. (10%)
10. Which of the following statements are true? (5%)
    (a) $\emptyset \subseteq \{\emptyset\}$; (b) $\emptyset \in \{\emptyset\}$; (c) $\emptyset \subseteq \emptyset$; (d) $\emptyset \subset \emptyset$; (e) $\emptyset \subset \{\emptyset\}$ (f) $\emptyset \in \emptyset$.