Midterm 2 of Discrete Mathematics

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Note: Please list complete process of the calculation or the proof for each problem.

1. Given two sets A and B. Express $\overline{A-B}$ in terms of $\cup$ and $\overline{}$. (10%)

2. A professor has two dozen introductory textbooks on computer science and is concerned about their coverage of the topics (A) compilers, (B) data structures, and (C) operating systems. The following data are the numbers of books that contain material on these topics:

$$\begin{array}{rrr} |A| = 8 & |B| = 13 & |C| = 13 \\ |A \cap B| = 5 & |A \cap C| = 3 & |B \cap C| = 6 \\ |A \cap B \cap C| = 2 & & \end{array}$$

   (a) How many of the textbooks include material on exactly one of these topics? (5%)
   (b) How many do not deal with any of the topics? (5%)
   (c) How many have no material on compilers? (10%)

3. For any $n \in \mathbb{Z}$, $n \geq 0$, prove that $2^{2n+1}+1$ is divisible by 3. (10%)

4. Use mathematical induction to show

$$\binom{n+1}{3} = \sum_{i=2}^{n} \binom{i}{2} \text{ for } n \geq 2. \quad (10\%)$$

5. If p, q are primes, prove that $p|q$ if and only if $p = q$. (10%)

6. Find the greatest common divisor of 486 and 126, and express the result as a linear combination of these two integers. (15%)

7. Determine which of the following functions from $\mathbb{Z}$ to $\mathbb{Z}$ is one-to-one and onto? (5%)
   (a) $f(n) = |n|$ (b) $f(n) = n-1$ (c) $f(n) = n^2+1$ (d) $f(n) = n^3$ (e) $f(n) = \lceil n/2 \rceil$.

8. (a) Let $A = \{1, 2, 3, 4\}$ and $B = \{x, y\}$, how many onto functions are there from A to B? (10%)
   (b) Let $C = \{a, b, c, d, e, f\}$. Find the number of ways to distribute elements in C into 3 identical containers with no container left empty. (10%)

9. (The pigeonhole principle) Let $S = \{3, 7, 11, 15, 19, \ldots, 95, 99, 103\}$. How many elements must we select from S to insure that there will be at least two whose sum is 110? (10%)

10. (The pigeonhole principle) $n+1$ distinct integers are chosen from $1, 2, 3, \ldots, 2n$. Show that among the integers chosen there are two such that one of them is divisible by the other. (10%)