Economics and Computation (2026 Fall)

Assignment 01

Chuang-Chieh Lin

Department of Computer Science and Engineering
National Taiwan Ocean University

Instructions. Show the essential reasoning for every answer. In Part I, payoffs are listed in the order (Player 1, Player 2). In Part II, all deviations are unilateral, so the other players continue to use σ_{-i}.

Part I: Basic Game Theory

Consider the following two-player payoff-maximization game. Player 1 chooses a row and Player 2 chooses a column.

	Player 2		
Player 1	x	y	z
a	$(1, 2)$	$(2, 2)$	$(5, 1)$
b	$(4, 1)$	$(3, 5)$	$(3, 3)$
c	$(5, 2)$	$(4, 4)$	$(7, 0)$
d	$(2, 3)$	$(0, 4)$	$(3, 0)$

Problem 1. Find the strictly dominant strategy for Player 1. Justify why it strictly dominates every other strategy.

Problem 2. When Player 1 plays d, what is Player 2's best response? Briefly justify your answer.

Problem 3. Find a weakly dominant strategy for Player 2. Verify the required weak inequalities against every strategy of Player 1.

Problem 4. Find all pure-strategy Nash equilibria. Explain why each reported outcome is a mutual best response.

Problem 5. Find all Pareto-optimal outcomes. For every omitted outcome, identify an outcome that Pareto dominates it.

Part II: Pure Deviations in an MSNE

Consider a finite cost-minimization game

$$\big(N, (S_i)_{i\in N}, (C_i)_{i\in N}\big), \qquad N = \{1, 2, \ldots, k\},$$

where every S_i is finite. Let $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_k)$ be a mixed-strategy profile and let

$$\sigma_{-i} := \prod_{j\neq i} \sigma_j.$$

For every $\rho_i \in \Delta(S_i)$, define

$$\overline{C}_i(\rho_i, \sigma_{-i}) := \mathbb{E}_{\substack{a_i\sim\rho_i \\ s_{-i}\sim\sigma_{-i}}}\big[C_i(a_i, s_{-i})\big].$$

Let δ_{a_i} denote the degenerate mixed strategy that assigns probability one to the pure strategy a_i.

Problem 6. Prove that, for every $\tau_i \in \Delta(S_i)$,

$$\overline{C}_i(\tau_i, \sigma_{-i}) = \sum_{a_i\in S_i} \tau_i(a_i)\, \overline{C}_i(\delta_{a_i}, \sigma_{-i}).$$

Problem 7. Suppose that τ_i is a profitable mixed deviation, meaning that

$$\overline{C}_i(\tau_i, \sigma_{-i}) < \overline{C}_i(\sigma_i, \sigma_{-i}).$$

Prove that there exists some $a_i \in S_i$ with $\tau_i(a_i) > 0$ such that

$$\overline{C}_i(\delta_{a_i}, \sigma_{-i}) < \overline{C}_i(\sigma_i, \sigma_{-i}).$$

In other words, prove that every profitable mixed deviation contains at least one profitable pure deviation.

Problem 8. Show that σ is a mixed-strategy Nash equilibrium if and only if, for every player $i \in N$ and every pure strategy $a_i \in S_i$,

$$\mathbb{E}_{s\sim\sigma}[C_i(s)] \le \mathbb{E}_{s_{-i}\sim\sigma_{-i}}\big[C_i(a_i, s_{-i})\big].$$

Conclude that checking all pure unilateral deviations is sufficient when verifying whether σ is an MSNE.

Hint for Problems 7–8. Use Problem 6 and the fact that the expected cost of a mixed deviation is a weighted average of the expected costs of its component pure strategies.

Part III: Borda Count and Condorcet Winners

Five voters have the following strict preferences over the four outcomes A, B, C, and D:

$$\begin{aligned}
&\text{Voter 1:} \quad B \succ C \succ A \succ D,\\
&\text{Voter 2:} \quad B \succ D \succ C \succ A,\\
&\text{Voter 3:} \quad D \succ C \succ A \succ B,\\
&\text{Voter 4:} \quad A \succ D \succ B \succ C,\\
&\text{Voter 5:} \quad A \succ D \succ C \succ B.
\end{aligned}$$

Problem 9. Answer both questions below and show the relevant score calculations or pairwise majority comparisons.

(a) Under the Borda count rule, assign $3, 2, 1$, and 0 points to a voter's first-, second-, third-, and fourth-ranked outcomes, respectively. Calculate the total score of every outcome and identify the Borda winner.

(b) Determine whether a Condorcet winner exists. Justify your conclusion using all pairwise majority comparisons needed to establish the answer.