

## Assignment 4

Due date: 15 April 2021

TA: 林宏懌 E817 (13:10~14:00)

1. Find  $p(A)$  for  $p(x) = x^2 - 2x + 1$  and  $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ .

2. Find the inverse of  $\begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ .

3. Show that if  $A$ ,  $B$ , and  $A + B$  are invertible matrices with the same size, then

$$A(A^{-1} + B^{-1})B(A + B)^{-1} = I.$$

4. Show that if  $A$  is a square matrix such that  $A^k = 0$  for some positive integer  $k$ , then the matrix  $I - A$  is invertible and

$$(I - A)^{-1} = I + A + A^2 + A^3 + \cdots + A^{k-1}.$$

5. Let  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ . Compute  $A^2 - (a + d)A + (ad - bc)I$ .