**Assignment 10**
**Due date: 11 June 2021**
**TA: 林宏懌 E817 (Online; iClass due to 23:59)**

1. Determine whether the following polynomials span $P_2$.

   $\mathbf{p}_1 = 1 - x + 2x^2, \quad \mathbf{p}_2 = 3 + x,$
   $\mathbf{p}_3 = 5 - x + 4x^2, \quad \mathbf{p}_4 = -2 - 2x + 2x^2.$

2. Let $T_A : \mathbf{R}^2 \mapsto \mathbf{R}^3$ be multiplication by matrix $A$. Determine whether the vector $\mathbf{u} = (1, 1, 1)$ is in the span of $\{T_A(\mathbf{e}_1), T_A(\mathbf{e}_2)\}$, where $A = \begin{bmatrix} 0 & 2 \\ 1 & -2 \\ 1 & 0 \end{bmatrix}$.

3. Determine whether the following vectors are linearly independent or linearly dependent in $P_2$.

   $\mathbf{p}_1 = 2 - x + 4x^2, \quad \mathbf{p}_2 = 3 + 6x + 2x^2, \quad \mathbf{p}_3 = 2 + 10x - 4x^2$

4. Determine whether the following vectors are linearly independent or linearly dependent in $P_2$.

   $\mathbf{p}_1 = 1 + 3x + 3x^2, \quad \mathbf{p}_2 = x + 4x^2, \quad \mathbf{p}_3 = 5 + 6x + 3x^2, \quad \mathbf{p}_4 = 7 + 2x - x^2.$

5. Use the Wronskian to show that $\mathbf{f}_1 = 1, \mathbf{f}_2 = e^x, \mathbf{f}_3 = e^{2x}$ are linearly independent vectors in $C^\infty(-\infty, \infty)$.