

## Assignment 2

Due date: 29 March 2024

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1. Let  $A$  be an  $n \times n$  symmetric matrix. Show that  $2A^2 - 3A + 2I$  is symmetric.

2. Suppose that  $A = \begin{bmatrix} 1 & 2 & \cdots & n \\ 2 & 4 & \cdots & 2n \\ \vdots & \vdots & \ddots & \vdots \\ n & 2n & \cdots & n^2 \end{bmatrix}$ . Please Compute  $\text{tr}(A^\top A)$ .

3. If  $A, B$ , and  $A^{-1} + B^{-1}$  are invertible  $n \times n$  matrices, show that  $A^2B + AB^2$  is invertible.

4. Consider  $\mathbf{x} = [x_1 \ x_2 \ x_3]^\top \in \mathbb{R}^3$  and  $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ , compute  $\text{tr}(A\mathbf{x}\mathbf{x}^\top)$ .

5. Compute  $A^8$  where  $A = \begin{bmatrix} 1 & -3 \\ 1 & 1 \end{bmatrix}$ .