1. If $\mathbf{b}, \mathbf{x} \in \mathbb{R}^n$ and $\mathbf{b} \neq \mathbf{0}$, then prove or disprove that $T(\mathbf{x}) = \mathbf{x} + \mathbf{b}$ is a matrix transformation on $\mathbb{R}^n$.

2. If $A^{n+1} = \mathbf{0}$, then prove that $(I - A)^{-1} = I + A + A^2 + \cdots + A^n$..

3. Find the standard matrix A for the linear transformation $T : \mathbb{R}^3 \mapsto \mathbb{R}^3$ for which

$$T\left(\begin{bmatrix} -2 \\ 3 \\ -4 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ 3 \\ 14 \end{bmatrix}, T\left(\begin{bmatrix} 3 \\ -2 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} -4 \\ 6 \\ -14 \end{bmatrix} \text{ and } T\left(\begin{bmatrix} -4 \\ -5 \\ 5 \end{bmatrix}\right) = \begin{bmatrix} -6 \\ -40 \\ -2 \end{bmatrix}.$$

4. Given that $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}$ and $\det(A) = 10$. Please compute the determinants for the following matrices.

   a. $\det(-3A)$

   b. $\det((2A^{-1})^\top)$

   c. $\det\left(\begin{bmatrix} a_3 & a_1 & a_2 \\ a_6 & a_4 & a_5 \\ a_9 & a_7 & a_8 \end{bmatrix}\right)$