Assignment 5
Due date: 29 May 2024
TA: 盧禹丞 E814

1. (20%) If $b \in \mathbb{R}$, then show that $\{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \mid x_3 = 5x_4 + b\}$ is a subspace of $\mathbb{R}^4$ if and only if $b = 0$.

2. (20%) Show that $V = \{(x, y, z) \in \mathbb{R}^3 \mid x + y + z = 0\}$ is a subspace of $\mathbb{R}^3$.

3. (20%) Let $T_A : \mathbf{R}^2 \mapsto \mathbf{R}^3$ be multiplication by matrix *A*. Determine whether the vector $\mathbf{u} = (1, 1, 1)$ is in the span of $\{T_A(\mathbf{e}_1), T_A(\mathbf{e}_2)\}$, where $A = \begin{bmatrix} 0 & 2 \\ 1 & -2 \\ 1 & 0 \end{bmatrix}$.

4. (20%) Determine whether the following vectors are linearly independent or linearly dependent in P_2.

   $\mathbf{p}_1 = 2 - x + 4x^2, \quad \mathbf{p}_2 = 3 + 6x + 2x^2, \quad \mathbf{p}_3 = 2 + 10x - 4x^2$

5. (20%) Use the Wronskian to show that $\mathbf{f}_1 = 1, \mathbf{f}_2 = e^x, \mathbf{f}_3 = e^{2x}$ are linearly independent vectors in $C^\infty(-\infty, \infty)$.