Midterm Exam of Linear Algebra

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10:10 – 12:00, 20 April 2021; Room E413

Note: Cell phones and any calculator are forbidden.

Part I: True (T) or False (F) (60%; each for 6%)

1. The plane $x + 4y - 2z = 9$ is a subspace of R^3.
2. $\{\boldsymbol{0}\}$ is a subspace of a vector space *V*.
3. $T(x, y) = (|x|, y)$ is a linear transformation.
4. It is possible for a system of linear equations to have exactly two solutions.
5. If *A* is an invertible symmetric matrix, then A^{-1} is symmetric.
6. Any symmetric matrix is invertible.
7. The product of invertible matrices is invertible.
8. For any two *n*-by-*n* matrices *A* and *B*, $(A + B)^2 = A^2 + 2AB + B^2$.
9. If *B* and *C* are both inverses of an *n*-by-*n* matrix *A*, then *B* = *C*.
10. If $B = R_2^{(1/2)} R_{12}^{(5)} R_{34} A$ where $R_2^{(1/2)}, R_{12}^{(5)}, R_{34}$ are three elementary matrices, then $A = R_{34} R_{12}^{(-5)} R_2^{(1/2)} B$.

Part II: Calculations. (60%; each for 12%)

1. Solve the system with variables x_1, x_2, x_3:

$$\begin{bmatrix} -1 & -2 & 3 & 1 \\ 1 & 1 & 2 & 8 \\ 3 & -7 & 4 & 10 \end{bmatrix}$$

2. Compute A^{20} where $A = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$

3. Compute the inverse of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$.

4. Let $A = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$. Find elementary matrices E_1 and E_2 such that $A = E_1 E_2$.

5. Find the standard matrix for the linear transformation $T : \mathbf{R}^2 \mapsto \mathbf{R}^2$ for which

$$T\left(\begin{bmatrix} -1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -5 \\ 5 \end{bmatrix} \text{ and } T\left(\begin{bmatrix} 2 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 7 \\ -6 \end{bmatrix}.$$