Mathematics for Machine Learning — Introduction

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Fall 2023

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Credits for the resource

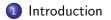
- The slides are based on the textbook:
 - Marc Peter Deisenroth, A. Aldo Faisal, and Cheng Soon Ong: Mathematics for Machine Learning. Cambridge University Press. 2020.
 - Howard Anton, Chris Rorres, Anton Kaul: Elementary Linear Algebra. Wiley. 2019.
- We could partially refer to the monograph: *Francesco Orabona: A Modern Introduction to Online Learning. https://arxiv.org/abs/1912.13213*

Grading Policy

- Attendance (10%)
- Assignments & Quizzes (30%)
- Midterm Exam (30%)
 - 7 Nov. 2023.
- Final Exam (30%)
 - 26 Dec. 2023.

(Sorry; just after the Christmas)

Outline



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ML Math Introduction

Three Core Concepts of Machine Learning

- Data
- Model
- Learning

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Remark on the Data

• Machine learning is inherently data driven.

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• Machine learning is inherently data driven.

• Garbage in, garbage out.

Mitchell 1997

A model is said to learn from data if its performance on a given task improves after the data is taken into account.

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Remark on the Data

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A model is said to learn from data if its performance on a given task improves after the data is taken into account.

• Goal: Find good models that generalize well to yet unseen data

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Four pillars of ML

The four pillars of ML:

- Regression
- Dimensionality Reduction
- Density Estimation
- Classification

Fundamentals:

- Calculus
- Linear Algebra
- Vector Algebra
- Analytic Geometry
- Matrix Decomposition
- Probability & Distributions
- Optimization

• Why are the mathematical foundations of machine learning important?

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- Why are the mathematical foundations of machine learning important?
 - To understand fundamental principles upon which more complicated machine learning systems are built.
 - To facilitate creating new machine learning solutions, understanding and debugging existing approaches.
 - To learn about the inherent assumptions and limitations of the methodologies we are working with.

What's a machine learning algorithm?

- Predictor: A system that makes predictions based on input data.
- Training: a system that adapts some internal parameters of the predictor so that it performs well on future unseen input data.

• Numerical representation of the data:

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- Numerical representation of the data: vectors.
 - An array of numbers (CS view)

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 - An array of numbers (CS view)
 - An arrow with a direction and magnitude (physics view)

- Numerical representation of the data: vectors.
 - An array of numbers (CS view)
 - An arrow with a direction and magnitude (physics view)
 - An object that obeys addition and scaling (mathematical view; OOP view).

An Intuition of Learning/Training a Model

- Assume that we are given a dataset and a suitable model.
- Training a model: use the data to optimize parameters of the model w.r.t. some loss/utility function.
- The training process can be viewed as either climbing a hill to reach its peak moving downwards to the valley.

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An Intuition of Learning/Training a Model

- Assume that we are given a dataset and a suitable model.
- Training a model: use the data to optimize parameters of the model w.r.t. some loss/utility function.
- The training process can be viewed as either climbing a hill to reach its peak moving downwards to the valley.
- However, at the same time, we are interested in the model which performs well on unseen data.

Otherwise, it could be just that we find a way to memorize the data.

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Part I.

Mathematics as the Foundation

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• The study of vectors and matrices.

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 - Analytic geometry (distance, norm, inner product, projection, ...)

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- The study of vectors and matrices.
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- Represent a table of data as a matrix.
- Formalize the *similarity* between vectors:
 - Analytic geometry (distance, norm, inner product, projection, ...)
- Intuitive interpretation of the data and better efficiency for learning: *matrix decomposition*.

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Part II:

Introductory Machine Learning

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Topics

- Data, model & parameter estimation.
- Continuous Optimization.
- Linear regression.
 - Map the input $\mathbf{x} \in \mathbb{R}^d$ to corresponding observed function values $y \in \mathbb{R}$.
- Density estimation.
 - Find a probability distribution that describes the data.
- Principal Component Analysis
 - Matrix decomposition.
- Classification.

Terminologies

- i.e. \implies that is,
- e.g. \implies such as
- \therefore \Longrightarrow because
- $\therefore \Longrightarrow$ therefore
- et al. \Longrightarrow and others
- $\forall \Longrightarrow$ for any
- $\exists \Longrightarrow$ there exists
- a.k.a. \implies also known as
- w.r.t. \implies with respect to
- i.i.d. \implies identically and independently distributed

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Warm-up Exercise

Exercise

• Consider
$$\mathbf{x} = [x_1 \ x_2 \ x_3]^\top \in \mathbb{R}^3$$
 and $\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$.
• Compute $\mathbf{x}^\top \mathbf{A} \mathbf{x}$.
• Compute $\operatorname{tr}(\mathbf{A} \mathbf{x} \mathbf{x}^\top)$.

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Reminders

- This is NOT a course of pure mathematics. This is also for ENGINEERING purpose!
- This is a course which can help you build solid foundation for machine learning (for both industrial and academical tasks and jobs).
- Preview before classes and Review after classes are strongly recommended.
- Absolute grades will be given; no final adjustment.

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Discussions

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