

Mathematics for Machine Learning (2025 Fall)

Assignment 01*

Chuang-Chieh Lin

Department of Computer Science and Engineering,
National Taiwan Ocean University

1. (20%) Consider $\mathbf{x} = [-2 \ 1 \ 1]^\top$ and the following matrix A , compute $\text{tr}(A\mathbf{x}\mathbf{x}^\top)$.

$$A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & 2 \\ 2 & 2 & 3 \end{bmatrix}.$$

2. (20%) Show that $\|\mathbf{v}\|_\infty \leq \|\mathbf{v}\|_1 \leq \sqrt{n}\|\mathbf{v}\|_2$ for any $\mathbf{v} \in \mathbb{R}^n$ (*Hint*: Use Cauchy-Schwarz Inequality).
3. (20%) Consider the following two matrices $A_1 = \begin{bmatrix} 9 & 6 \\ 6 & 5 \end{bmatrix}$, $A_2 = \begin{bmatrix} 9 & 6 \\ 6 & 3 \end{bmatrix}$. Prove the following arguments:
- (a) A_1 is positive definite.
 - (b) A_2 is not positive definite
4. (20%) Prove that for any $A \in \mathbb{R}^{m \times n}$, $B \in \mathbb{R}^{n \times p}$, $m, n, p \in \mathbb{N}$, we have $\text{tr}(AB) = \text{tr}(BA)$.
5. (20%) For the identity matrix $I_n \in \mathbb{R}^{n \times n}$,
- (a) What are its eigenvalues and the associated eigenvectors?
 - (b) What are the eigenspaces?

* List the required intermediate steps next to each problem. Note that any answers generated directly by AI are invalid for this assignment.