Mathematics for Machine Learning (2025 Fall)

Assignment 03*

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1. (20%) Let $\boldsymbol{A} \in \mathbb{R}^{n \times n}$ and $\mathbf{x} \in \mathbb{R}^{n \times 1}$. Show that

$$\mathbb{E}_X[\boldsymbol{A}(\mathbf{x})] = \boldsymbol{A}\,\mathbb{E}_X[\mathbf{x}].$$

2. (20%) Given $Y \sim \mathcal{N}(\boldsymbol{\mu}_y, \boldsymbol{\Sigma})$ and $\mathbf{y} = \boldsymbol{A}\mathbf{x}$ for $\mathbf{x}, \mathbf{y} \in \mathbb{R}^M$, and $\boldsymbol{A}$ is invertible. Suppose that $p(\mathbf{y}) = \mathcal{N}(\mathbf{y} \mid \boldsymbol{A}\mathbf{x}, \boldsymbol{\Sigma})$. Please compute the distribution of X.

3. (20%) Given $\mathbf{x} \in \mathbb{R}^n$, please compute $\frac{\mathrm{d}}{\mathrm{d}\mathbf{x}}(2\mathbf{x})$.

4. (20%) Suppose that X is a continuous random variable with pdf $f_X : [0,1] \to [0,1]$ such that $f_X(x) = 3x^2$. Show that $Y := F_X(X)$ attains a uniform distribution.

5. (20%) Given $\mathbf{x}, \mathbf{y}, \mathbf{b} \in \mathbb{R}^n, \mathbf{A} \in \mathbb{R}^{n \times n}$, if $\mathbf{x}, \mathbf{y}$ are random vectors such that $\mathbf{y} = 2\boldsymbol{A}\mathbf{x} - \mathbf{b}$ and $\mathbb{V}[\mathbf{x}] = \boldsymbol{\sigma}$, please compute the variance $\mathbb{V}[\mathbf{y}]$.

* List the required intermediate steps next to each problem. Note that any answers generated directly by AI are invalid for this assignment.