

Randomized Algorithms (2026 Spring)

Assignment Set 1*

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1. Suppose a randomly chosen person was born on Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, or Saturday with equal probability. If we select n people uniformly at random, then:
 - (a) What is the least value of n to guaranty that at least two of the chosen people were born on the same weekday?
 - (b) What is the least value of n to guaranty that at least two of the chosen people were born on the same weekday with a probability of at least $1/2$?

2. Prove that $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{i}{2^i} \leq 2$.

3. Please explain why we can't use Karger's randomized edge-contraction algorithm to solve the "max-cut" problem.

4. Please show that

$$\left(1 - \frac{1}{n^3}\right)^T \leq \frac{1}{n},$$

where $T = n^3 \ln n$.

5. There are 100 people going to a party and leaving their hats at the door. At the end of the night, the host is a bit tired and hands the hats back completely at random. **On average, how many people will get their own hat back?**
6. In the **Rand- $(k$ -Select** algorithm, we pick a number y uniformly at random in an array S of n distinct numbers and compute $S_1 := \{x \in S \mid x < y\}$, $S_2 := \{x \in S \mid x > y\}$. Let $X := \max\{|S_1|, |S_2|\}$. Please prove that $\mathbb{E}[X] \leq 3n/4$.
7. Given a language L . There exists a randomized algorithm A running in worst-case polynomial time such that for any input $x \in \Sigma^*$, $n := |x|$, we have $\Pr[A(x) \text{ accepts}] \geq \frac{3}{5}$ if $x \in L$, and $\Pr[A(x) \text{ rejects}] \geq \frac{3}{5}$ if $x \notin L$. Please prove that $L \in \mathbf{BPP}$ (i.e., please propose an algorithm with successful probability $\geq 3/4$ for both cases).
8. A coin x is called a p -coin if it shows HEAD for one coin flip with a probability of p . Please prove that we can transform x to a fair-coin (i.e., the coin showing head with probability $1/2$) in expected polynomial time.

* We will select five of them for the Quiz.