Testing $K_{3,3}$-freeness in dense graphs:

an example of property testing with a two-sided error and a one-sided error

Noga Alon and Asaf Shapira

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Speaker: Joseph, Chuang-Chieh Lin
Supervisor: Professor Maw-Shang Chang

Computation Theory Laboratory
Department of Computer Science and Information Engineering
National Chung Cheng University, Taiwan

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Outline

1. Introduction
2. Testing $K_{t,t}$-freeness in dense graphs with a two-sided error
3. Testing $K_{t,t}$-freeness in dense graphs with a one-sided error
4. Conclusions

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Background of property testing

- In the real world nowadays, we are faced with imperious need to process increasing larger amounts of data in faster times.
- Many practical problems have inputs of very large size.
- Sometimes it is not realistic to solve a problem in the time even linear in the input size.
- Property testing is one of the possible approaches faster than linear time algorithms.

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Background of property testing (contd.)

- Try to answer "yes" or "no" for the following *relaxed* decision problems by observing only a small fraction of the input.
  - Does the input **satisfy a designated property**, or
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Background of property testing (contd.)

- In property testing, we use ϵ-far to say that the input is far from a certain property.
- ϵ: the least fraction of the input needs to be modified.
- For example:
  - A sequence of integers $L = (0, 2, 3, 4, 1)$.
  - Allowed operations: integer deletions
  - L is 0.2-far from being monotonically nondecreasing.

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The model for dense graphs

- Graph representation: adjacency-matrix for a graph $G = (V, E)$.
  - undirected, no self-loops, ≤ 1 edge between any $u, v \in V$.
  - $|V| = n$ vertices and $|E| = \Omega(n^2)$ edges.
  - A query: to see if two vertices u and v are adjacent or not.

- ϵ-far from satisfying $\mathbb{P}$:
  - $\geq \epsilon n^2$ edges should be deleted or added to make G satisfy $\mathbb{P}$.

Background of property testing (contd.)

- The complexity measure: queries.
- In property testing, the query complexity (say $q(n, \epsilon)$) is asked to be sublinear **in the input size** (say $f(n)$).
  - $q(n, \epsilon) = o(f(n))$ if $\lim_{n \to \infty} \frac{q(n,\epsilon)}{f(n)} \to 0$, where ϵ is viewed as a constant.

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Property testers

- A property tester for $\mathbb{P}$ is an algorithm utilizing sublinear queries such that:
    - ▷ if the input satisfies $\mathbb{P}$:
      answers "yes" with probability $\geq 2/3$ ($1 \rightarrow$ one-sided error);
    - ▷ if the input is ϵ-far from satisfying $\mathbb{P}$:
      answers "no" with probability $\geq 2/3$.

Goals of this talk

- The property: $K_{t,t}$-freeness.
  - A graph is $K_{t,t}$-free if it does not have a $K_{t,t}$ as a subgraph.

Fig.: $K_{2,2}$ and $K_{3,3}$.

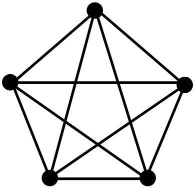

Fig.: K_5 is not $K_{2,2}$-free.

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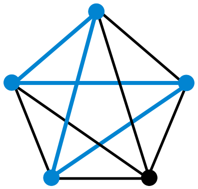

Fig.: K_5 is not $K_{2,2}$-free.

Goals of this talk (contd.)

- Reveal the difference between testing with a *two*-sided error and with a *one*-sided error through testing $K_{t,t}$-freeness in dense graphs.

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$K_{t,t}$ and t-stars

- t-star: A star of size t (a vertex having t neighbors)

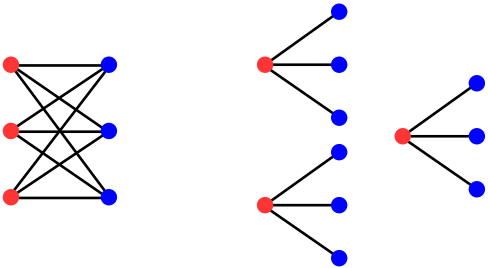

Fig.: $K_{3,3}$ and 3-stars.

- We say that t-star with edges $(a, b), (a, c), (a, d)$ sits on $\{b, c, d\}$.

A large enough dense graph must contain a $K_{t,t}$ as its subgraph

Observation

If a graph contains ϵn^2 edges, then it contains at least $\frac{\epsilon}{2}n$ vertices of degree at least $\frac{\epsilon}{2}n$.

Proof.

The proof is omitted since it is easy. □

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Claim 1

For any ϵ and t, and for sufficiently large n, any graph with at least ϵn^2 edges contains a $K_{t,t}$ as a subgraph.

Proof.

- If G has ϵn^2 edges then it contains $\geq \epsilon n/2$ vertices of degree $\geq \frac{\epsilon}{2} n$ (by the previous observation).
- The number of t-stars in G is at least

$$\frac{\epsilon}{2} n \binom{\frac{\epsilon}{2} n}{t} \geq \frac{\epsilon}{2} n \left(\frac{\epsilon n}{2t}\right)^t \geq \left(\frac{\epsilon n}{2t}\right)^{t+1} > tn^t > \binom{n}{t}(t-1).$$

- By the pigeonhole principle, there must be at least one set of t vertices where at least 't' t-stars are sitting.

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- Assume that n is sufficiently large.
- Consider the algorithm as follows.

A two-sided error tester

1. Select $100/\epsilon$ pairs of vertices uniformly at random. Denote the set of these pairs by S.

2. For $(u, v) \in S$, query whether (u, v) is an edge of G.

3. $\delta \leftarrow |\{(u, v) \mid (u, v) \in S,\ (u, v) \text{ is an edge of } G\}|$.

    △ If $\delta \leq 100$, return "yes";

    △ Otherwise, return "no".

A two-sided error tester (contd.)

- By the previous claim, if G contains more than $\frac{1}{4}\epsilon n^2$ edges, then it contains a $K_{t,t}$.
- If G contains less than ϵn^2 edges, it is trivially not ϵ-far from being $K_{t,t}$-free.

A two-sided error tester (contd.)

- If G is $K_{t,t}$-free, then it contains at most $\frac{1}{4}\epsilon n^2$ edges. Thus by Chebyshev's inequality, we have

$$\begin{aligned} & \mathbf{Pr}[\delta > 100] \leq \mathbf{Pr}[\delta \geq 100] \\ \leq \quad & \mathbf{Pr}[|\delta - 50| \geq 50] \\ \leq \quad & \frac{(100/\epsilon) \cdot \left(\frac{\epsilon n^2/4}{n(n-1)/2}\right) \cdot (1-\epsilon/2)}{50^2} \\ < \quad & \frac{1}{3}. \end{aligned}$$

- If G is ϵ-far from being $K_{t,t}$-free, then it contains at least ϵn^2 edges. Thus also by Chebyshev's inequality, we have

$$\mathbf{Pr}[\delta \leq 100] \leq \mathbf{Pr}[|\delta - 200| \geq 100] \leq \frac{200(1-2\epsilon)}{100^2} < \frac{1}{3}.$$

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Theorem 1

For sufficiently large n, testing $K_{t,t}$-freeness with a two-sided error in an n-vertex dense graph can be done by using at most $O(1/\epsilon^2)$ queries.

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Next, we are going to show the existence of a graph G, where

- G is ϵ-far from being $K_{t,t}$-free, and
- more than half of the induced subgraphs of size $(1/8\epsilon)^{t/2}$ do not contain a $K_{t,t}$.

A random graph $G(n, 4\epsilon)$ helps!

- Consider a random graph $G(n, 4\epsilon)$
  - a graph on n vertices
  - each pair of vertices is an edge with probability 4ϵ.
- Fix a set T of $2t$ vertices, then

$$\mathbf{Pr}[T \text{ contains a } K_{t,t}] \leq \binom{2t}{t}(4\epsilon)^{t^2}.$$

- Thus the expected number of copies of $K_{t,t}$ is at most

$$\binom{n}{2t}\binom{2t}{t}(4\epsilon)^{t^2}.$$

- Event A: the number of copies of $K_{t,t}$ in $G(n, 4\epsilon)$ is **more than twice the expectation**.
- Event B: $G(n, 4\epsilon)$ contains **less than $\frac{3}{2}\epsilon n^2$ edges**.

- What are $\mathbf{Pr}[A]$ and $\mathbf{Pr}[B]$?
  - $\mathbf{Pr}[A] < \frac{1}{2}$ (by Markov's inequality).
  - $\mathbf{Pr}[B] < \frac{1}{2}$ (by the Chernoff bound).
    - $1.98\epsilon n^2 < \mathbf{E}[X] = 4\epsilon\binom{n}{2} < 2\epsilon n^2$ (X: the number of edges of G; $n > 100$ by the assumption that n is sufficiently large)
    - Let $\mu = 1.98\epsilon n^2$, then

$$\begin{aligned}\mathbf{Pr}[X \leq \frac{3}{2}\epsilon n^2] &= \mathbf{Pr}\left[X \leq \left(1 - \frac{0.48}{1.98}\right)\mu\right] \\ &\leq e^{-\mu\cdot(0.48/1.98)^2/2} < e^{-0.1\epsilon n^2} \\ &< \frac{1}{2}.\end{aligned}$$

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What we concern about is ...

- $\mathbf{Pr}[\overline{A} \cap \overline{B}] = \mathbf{Pr}[\overline{A \cup B}] = 1 - \mathbf{Pr}[A \cup B] > 0.$
- That is, there exists at least one graph G' that **has no more than twice the expected number of copies of $\mathbf{K_{t,t}}$** and **is ϵ-far from being $\mathbf{K_{t,t}}$-free!**
- The fraction of sets of size $2t$ that contain a $K_{t,t}$ in G' is at most

$$\frac{2\binom{n}{2t}\binom{2t}{t}(4\epsilon)^{t^2}}{\binom{n}{2t}} = 2\binom{2t}{t}(4\epsilon)^{t^2} \leq 2 \cdot 4^t (4\epsilon)^{t^2} \leq 2(8\epsilon)^{t^2}.$$

- If we choose a random set of vertices S of size $(1/8\epsilon)^{t/2}$, it contains

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- Thus the probability that the randomly chosen set of vertices S contains a copy of $K_{t,t}$ is less than $\frac{1}{4}\left(\frac{1}{8\epsilon}\right)^{t^2} 2(8\epsilon)^{t^2} = \frac{1}{2}$.

Theorem 2

For any sufficiently large n and $\epsilon < 1/4$, there is an n-vertex graph G, that is ϵ-far from being $K_{t,t}$-free, such that most induced subgraphs, on sets of size $(1/8\epsilon)^{t/2}$, do not contain a $K_{t,t}$ as a subgraph.

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- To work on property testing with one-sided error is somehow more difficult, but is very common!
- Sometimes we can devise a property tester for another property which is "close" to the original one.

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  - Markov's inequality, Chebyshev's inequality, Chernoff bounds, Bernoulli and Binomial distributions, random graphs, the probabilistic method, basic counting skills.
- Actually, to work on graph property testing, we just need to **understand the graphs more!** Probability theory is not the most important thing.
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不
這是唯一的方法了


你是我這個黑鬼
世上唯一好友


假設我們只能存活一人，那就選擇你吧


感謝你陪我
度過每個美好的日子
嗚
嗚


嘟

嗚
嗚

Thank you!