Of the People: Voting Is More Effective with Representative Candidates

Yu Cheng, Shaddin Dughmi, David Kempe

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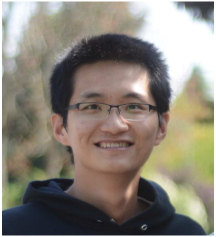
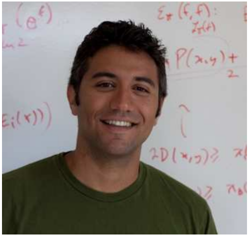


Yu Cheng



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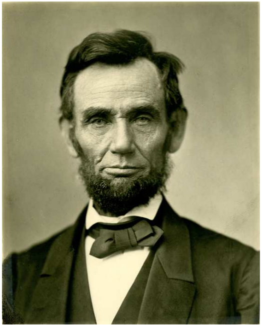
My CV

Publications

- *Finding Best Mixture of Nash is Hard.*
  Yu Cheng, Young Kun Ko.
- *Of the People: Voting Is More Effective with Representative Candidates.* (arXiv)
  Yu Cheng, Shaddin Dughmi, David Kempe. **EC 2017.**
- *Well-Supported versus Approximate Nash Equilibria: Query Complexity of Large Games.* (arXiv)
  Xi Chen, Yu Cheng, Bo Tang. **ITCS 2017.**
- *Playing Anonymous Games using Simple Strategies.* (arXiv)
  Yu Cheng, Ilias Diakonikolas, Alistair Stewart. **SODA 2017.**
- *On the Recursive Teaching Dimension of VC Classes.* (ECCC)
  Xi Chen, Yu Cheng, Bo Tang. **NIPS 2016.**
- *Hardness Results for Signaling in Bayesian Zero-Sum and Network Routing Games.* (arXiv)
  Umang Bhaskar, Yu Cheng, Young Kun Ko, Chaitanya Swamy. **EC 2016.**
- *Mixture Selection, Mechanism Design, and Signaling* (arXiv)
  Yu Cheng, Ho Yee Cheung, Shaddin Dughmi, Ehsan Emamjomeh-Zadeh, Li Han, Shang-Hua Teng. **FOCS 2015.**
- *Efficient Sampling for Gaussian Graphical Models via Spectral Sparsification* (arXiv Part I, Part II)
  Dehua Cheng, Yu Cheng, Yan Liu, Richard Peng, Shang-Hua Teng. **COLT 2015.**
- *Signaling in Quasipolynomial Time* (arXiv)
  Yu Cheng, Ho Yee Cheung, Shaddin Dughmi, Shang-Hua Teng.



"[. . .] and that government of the people, by the people, for the people, shall not perish from the earth."

— Abraham Lincoln, 1863.

The central question in this paper:

> If a government by the people is to be for the people, how important is it that it also be of the people?

A mapping of Lincoln's vision onto central concepts of the *social choice theory*:

- Who is the government **of**?
  - Who are the candidates to be aggregated?
- Who is the government **by**?
  - What are the social choice rules used for aggregation?
- Who is the government **for**?
  - What objective function is to be optimized?

Outline

Social choice rules

- Voters provide an ordinal ranking of (a subset of) the candidates,
- Aggregate these rankings to produce either a singler winner or a consensus ranking of all (or some) candidates.

Gibbard–Satterthwaite Theorem (1973)

Given a deterministic electoral system that choose a single winner. For every voting rule, one of the following three things must hold:

- The rule is dictatorial.
- The rule limits the possible outcomes to two alternatives only.
- The rule is susceptible to tactical voting.

Circumventing the impossibility of social choice

- A natural model: embedding candidates & voters in a metric space.
  - Small distances (cost) model high agreement (utility).
  - Introducing a preference order over candidates for each voter.
  - Providing an objective function naturally:
    - ⋆ the best alternative: the one closest to the voters on average.
- Circumventing the impossibility of social choice through approximation.
  - The distortion:
  - ⋆ How much worse is the outcome of voting than would be the omniscient choice of the best available candidate?

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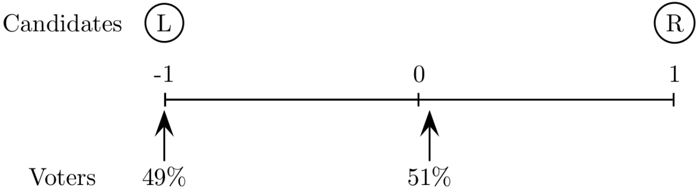
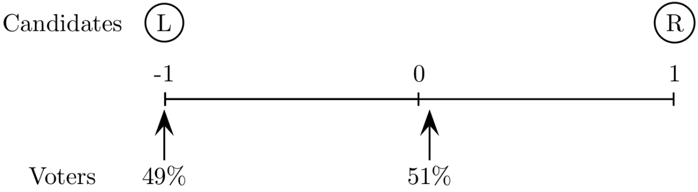


- The average distance from the population to candidate L: ≈ 0.5.
- The average distance from the population to candidate R: ≈ 1.5.
- But R will be elected as the winner in the election.



- Intuitively, when candidates are drawn from the population, we would expect the distortion in the social cost to be better than they are not.

Preliminaries

- Problem instance: $(\mathcal{D}, \boldsymbol{p}, \boldsymbol{q})$.
  - $d_{i,j}$: the distance between i and j in the metric space.
  - $\mathcal{D} = (d_{i,j})_{i,j}$: the finite metric space.
  - $\boldsymbol{p} = (p_i)_i$: the candidate distribution.
  - $\boldsymbol{q} = (q_i)_i$: the voter distribution.
- $c_i = \sum_j q_j \cdot d_{i,j}$: the social cost of candidate i (i.e., the average distance to all voters).

⋆ When candidates i and i' are competing, each voter j votes for $\arg\min_{i,i'}\{d(j,i), d(j,i')\}$.

- $w(i,i')$: the winner;
  - ⋆ i wins iff $\sum_{j:d_{i,j} \leq d_{i',j}} \geq 1/2$.
- $o(i,i') = \arg\min_{j \in \{i,i'\}} c_j$: the candidate of lower social cost.

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Distortion

- The distortion b/w i, i':

$$r_{i,i'} = \frac{c_{w(i,i')}}{c_{o(i,i')}}.$$

- We are interested in the expected distortion of the instance $(\mathcal{D}, \boldsymbol{p}, \boldsymbol{q})$:

$$C(\mathcal{D}, \boldsymbol{p}, \boldsymbol{q}) = \mathbf{E}_{i,i' \sim \boldsymbol{p}}[r_{i,i'}] = \mathbf{E}_{i,i' \sim \boldsymbol{p}}\left[\frac{c_{w(i,i')}}{c_{o(i,i')}}\right]$$
$$= 2\sum_{i<i'} p_i p_{i'} \cdot \frac{c_{w(i,i')}}{c_{o(i,i')}} + \sum_i p_i^2 \cdot 1.$$

Contribution of this paper

When $\boldsymbol{p} = \boldsymbol{q}$:

- On the simplest metric space: the line (political spectrum):
  - $\max_{\mathcal{D},\boldsymbol{p}} C(\mathcal{D}, \boldsymbol{p}, \boldsymbol{p}) = 4 - 2\sqrt{2} \approx 1.1716$ (tight).
- On the general metric space:
  - $\max_{\mathcal{D},\boldsymbol{p}} C(\mathcal{D}, \boldsymbol{p}, \boldsymbol{p}) \in (\frac{3}{2},\ 2 - \frac{1}{652}]$.

When $\boldsymbol{p} \neq \boldsymbol{q}$:

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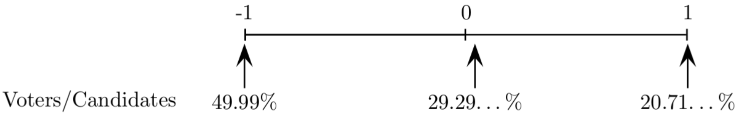
On identical distributions on the line

The lower bound

$p_1 = \frac{1}{2} - \epsilon$ voters @ $x_1 = -1$, $p_2 = 1 - \frac{1}{\sqrt{2}}$ voters @ $x_2 = \epsilon$, $p_3 = \frac{1}{\sqrt{2}} - \frac{1}{2} + \epsilon$ voters @ $x_3 = 1$.



- $c_1 = p_2 d_{1,2} + p_3 d_{1,3} = p_2 + 2p_3 + O(\epsilon) = \frac{1}{\sqrt{2}} + O(\epsilon)$.
- $c_3 = p_1 d_{1,3} + p_2 d_{2,3} = 2p_1 + p_2 - O(\epsilon) = 2 - \frac{1}{\sqrt{2}} - O(\epsilon)$.
- $c_2 = p_1 d_{1,2} + p_3 d_{2,3} = \frac{1}{\sqrt{2}}$.

⋆ $C(\mathcal{D}, \boldsymbol{p}, \boldsymbol{p}) = (1 - 2p_1 p_3) \cdot 1 + (2p_1 p_3) \cdot \frac{c_3}{c_1} = 4 - 2\sqrt{2} - O(\epsilon)$.

The upper bound (line)

Theorem 3

Let the underlying metric space be the line. For any distribution $\boldsymbol{p}$, we have $C(\mathcal{D}, \boldsymbol{p}, \boldsymbol{p}) \leq 4 - 2\sqrt{2}$.

Characterizing the structure of voting on the line

Given a distribution $\boldsymbol{p}$ one the line with support size n, we label the support points as $1, \ldots, n$.

- $m \triangleq$ the index of the median, $L \triangleq \{1, \ldots, m-1\}$ and $R \triangleq \{m+1, \ldots, n\}$.
  - ⋆ $p_L < 1/2 < p_L + p_m$ and $p_R < 1/2 < p_m + P_R$.

Lemma 4

If two candidates (x, y) are drawn, the one closer to m wins the election.

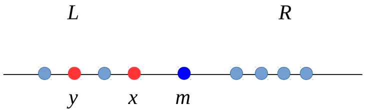
Lemma 5

If x, y are on the same side of the median m (including one of them being the median), then the one closer to m has smaller social cost.

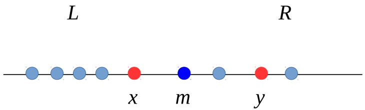
Lemma 4

If two candidates (x, y) are drawn, the one closer to m wins the election.

- WLOG, assume $d_{x,m} < d_{y,m}$ and $x \in L \cup \{m\}$.
- If $y \in L$:



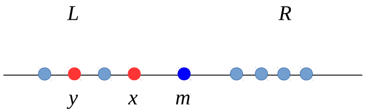
- If $y \in R$:



Lemma 5

If x, y are on the same side of the median m (including one of them being the median), then the one closer to m has smaller social cost (i.e., $c_x \leq c_y$ if $d_{x,m} < d_{y,m}$).

- Intuitively, x has smaller social cost because $> 1/2$ of the population need to first get to x before getting to y.



$$
\begin{aligned}
c_x &= \sum_{i \in L} p_i d_{i,x} + \sum_{i \in \{m\} \cup R} p_i d_{i,x} = \sum_{i \in L} p_i d_{i,x} + \sum_{i \in \{m\} \cup R} p_i (d_{i,y} - d_{x,y}) \\
&\leq \sum_{i \in L} p_i (d_{i,x} - d_{x,y}) + \sum_{i \in \{m\} \cup R} p_i d_{i,y} \\
&\leq \sum_{i \in L} p_i d_{i,y} + \sum_{i \in \{m\} \cup R} p_i d_{i,y} = c_y
\end{aligned}
$$

- By Lemmas 4 & 5, we can rewrite $C(\mathcal{D}, \boldsymbol{p}, \boldsymbol{p})$ as

$$C(\mathcal{D}, \boldsymbol{p}, \boldsymbol{p}) = \sum_{i \in [n]} p_i^2 + \sum_{i,j \in [n], i<j} 2 p_i p_j r_{i,j}$$
$$= 1 + \sum_{i \in L, j \in R} 2 p_i p_j (r_{i,j} - 1).$$

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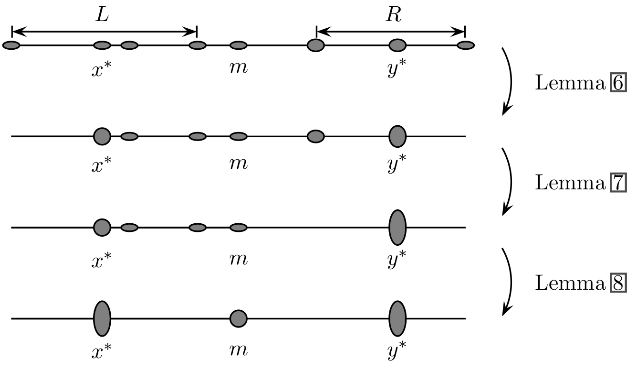
Proof of the upper bound $4 - 2\sqrt{2}$



- $r_i = \sum_j p_j r_{i,j}$: the expected distortion conditioned on one of the candidates being i.
- $y^* \triangleq \arg\max_{y \in R} r_y$, $x^* \triangleq \arg\max_{y \in L} r_x$

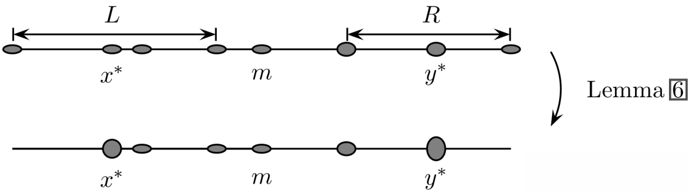
Proof of the upper bound $4 - 2\sqrt{2}$ (contd.)

- We can rewrite $C(\mathcal{D}, \boldsymbol{p}, \boldsymbol{p})$ as

$$C(\mathcal{D}, \boldsymbol{p}, \boldsymbol{p}) = 1 + \sum_{i \in L, j \in R} 2 p_i p_j (r_{i,j} - 1)$$
$$= 1 + 2 \sum_{i \in L, j \in [n]} p_i p_j (r_{i,j} - 1)$$
$$= 1 - 2p_L + 2 \sum_{i \in L} p_i r_i,$$

or

$$C(\mathcal{D}, \boldsymbol{p}, \boldsymbol{p}) = 1 - 2p_R + 2 \sum_{i \in R} p_i r_i.$$



- For all $1 \leq i, j \leq y^*$,

$$r'_{i,j} = \frac{c'_{w(i,j)}}{c'_{o(i,j)}} = \frac{c_{w(i,j)} - \sum_{y>y^*} p_y d_{y,y*}}{c_{o(i,j)} - \sum_{y>y^*} p_y d_{y,y*}} \geq \frac{c_{w(i,j)}}{c_{o(i,j)}} = r_{i,j}.$$

Proof of the upper bound $4 - 2\sqrt{2}$ (contd.)

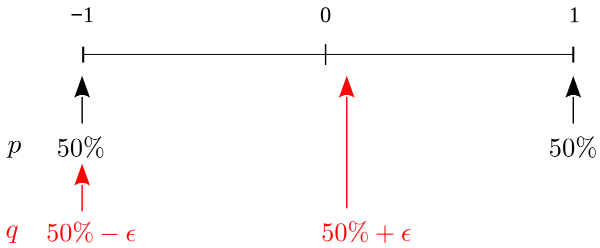
- By Lemmas 6, 7, and 8, the worst-case instance has support size ≤ 3.
- WLOG, let $x_1 = 0, x_3 = 1, x_2 > 1/2$.
- If $x_2 \neq m$ (not the median), the socially better candidate would always win (i.e., $C(\mathcal{D}, \boldsymbol{p}, \boldsymbol{p}) = 1$).
- If x_2 is the median, since $x_2 > 1/2$, x_3 wins x_1 since $d_{x_3,m} < d_{x_1,m}$.
  - For the worst-case, x_1 must have lower cost than x_3.
- Then we have

$$C(\mathcal{D}, \boldsymbol{p}, \boldsymbol{p}) = (1 - 2p_1p_3) \cdot 1 + 2p_1p_3 \cdot \frac{c_3}{c_1}$$
$$= (1 - 2p_1p_3) + 2p_1p_3 \cdot \frac{p_1 + p_2(1 - x_2)}{p_2x_2 + p_3}.$$

- It is maximized by taking $x_2 \to \frac{1}{2}$, $p_1 \to \frac{1}{2}$.
- $C(\mathcal{D}, \boldsymbol{p}, \boldsymbol{p}) \leq (1 - p_3) + p_3 \cdot \frac{3-2p_3}{p_2/2+p_3} \leq 4 - 2\sqrt{2}$, maximized at $p_3 = \frac{\sqrt{2}-1}{2}$.

Distortion for different distributions $\boldsymbol{p} \neq \boldsymbol{q}$

A lower bound



- With prob. 1/2, we draw two different candidates → distortion $3 - O(\epsilon)$.
- With prob. 1/2, we draw two candidates from the same location → distortion 1.

⋆ The expected distortion: $2 - O(\epsilon) \to 2$ as $\epsilon \to 0$.

The upper bound

Theorem 9

For all instances $(\mathcal{D}, \boldsymbol{p}, \boldsymbol{q})$, we have $C(\mathcal{D}, \boldsymbol{p}, \boldsymbol{q}) \leq 2$.

Required Lemmas for Theorem 9

Lemma 10

Let $i = w(i, i')$. Then $c_i \leq 3c_{i'}$.

Main ideas:

- i beats $i' \Rightarrow \geq 50\%$ voters are at least close to i as to i';
- For any j who is at least close to i as to i', we have $d_{i',i} \leq d_{i',j} + d_{j,i} \leq 2d_{i',j}$ (triangular inequality).

Lemma 11

For any $1 \leq \alpha \leq 3$ and any instance $(\mathcal{D}, \boldsymbol{p}, \boldsymbol{q})$, if $r_{i,j} = \frac{c_{w(i,j)}}{c_{o(i,j)}} \leq \alpha$ for all (i, j), then $C(\mathcal{D}, \boldsymbol{p}, \boldsymbol{q}) \leq \frac{1+\alpha}{2}$.

Proof of Lemma 11

- Consider an instance $(\mathcal{D}, \boldsymbol{p}, \boldsymbol{q})$ and its associated cost $\boldsymbol{c}$.
  - WLOG, assume that $c_1 \leq c_2 \leq \ldots \leq c_n$.
- For each candidate i, let $\ell_i \triangleq \max\{j \mid c_j \leq \alpha c_i\}$.
- Since $r_{i,j} \leq \alpha$ for all i, j, we have $w(i,j) = o(i,j)$ whenever $j > \ell_i$ (cost ratio $= 1$).

▷
$$C(\mathcal{D}, \boldsymbol{p}, \boldsymbol{q}) - 1 \leq 2 \sum_{i<j\leq\ell_i} p_i p_j \cdot \left(\frac{c_j}{c_i} - 1\right) \triangleq \hat{C}(\boldsymbol{p}, \boldsymbol{c}, \alpha).$$

⋆ $\hat{C}(\boldsymbol{p}, \boldsymbol{c}, \alpha)$ can be maximized by moving probability mass so that c_i and c_j are at most a factor α for every i and j in the support of $\boldsymbol{p}$.

Proof of Lemma 11 (contd.)

$$2 \sum_{i<j\leq \ell_i} p_i p_j \cdot \left(\frac{c_j}{c_i} - 1\right) \triangleq \hat{C}(\boldsymbol{p}, \boldsymbol{c}, \alpha).$$

- Suppose that there exists a pair $i < j$ in the support of $\boldsymbol{p}$ with $j > \ell_i$, i.e., $c_j > \alpha c_i$.
- Consider moving ϵ (or $-\epsilon$) prob. mass from p_i to p_j, call the resulting prob. vector $\boldsymbol{p}(\epsilon)$.
- Note that $\hat{C}(\boldsymbol{p}(\epsilon), \boldsymbol{c}, \alpha)$ is a linear function of ϵ (our choice of i, j avoids the bilinear term $p_i p_j$).
- The expression is maximized by moving all the prob. mass from one of i and j to the other.

Proof of Lemma 11 (contd.)

$$2 \sum_{i<j\leq \ell_i} p_i p_j \cdot \left(\frac{c_j}{c_i} - 1 \right) = 2 \sum_{i<j} p_i p_j \cdot \left(\frac{c_j}{c_i} - 1 \right) \triangleq \hat{C}(\boldsymbol{p}, \boldsymbol{c}, \alpha).$$

- Assume that support($\boldsymbol{p}$) is $n' \geq 3$, and associated costs are $c_1 < c_2 < \ldots < c_{n'}$.
- Consider all terms except c_2 as constants, then $\hat{C}(\boldsymbol{p}, \boldsymbol{c}, \alpha)$ is of the form $\beta_1 + \beta_2 c_2 + \beta_3 / c_2$, with $\beta_2, \beta_3 \geq 0$, which is convex in c_2.
- It attains the maximum at $c_2 = c_1$ or $c_2 = c_3$.
  - In either case, we can merge the prob. mass of point 2 with 1 or 3, reducing the support size by 1 without decreasing $\hat{C}(\boldsymbol{p}, \boldsymbol{c}, \alpha)$.
  - By repeating such merges, we eventually arrive at a distribution with support size 2 and $c_2 \leq \alpha c_1$.
- Finally, we have

$$\begin{aligned} C(\mathcal{D}, \boldsymbol{p}, \boldsymbol{q}) = 1 + \hat{C}(\boldsymbol{p}, \boldsymbol{c}, \alpha) &\leq 1 + 2p_1(1 - p_1) \cdot (\alpha - 1) \\ &\leq 1 + \frac{1}{2}(\alpha - 1) = \frac{1 + \alpha}{2}. \end{aligned}$$

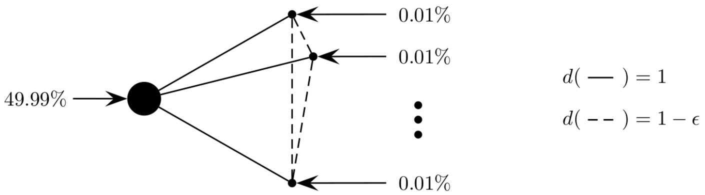
On identical distributions in general metric spaces

Theorem 12

The worst-case distortion $\sup_{(\mathcal{D},\boldsymbol{p},\boldsymbol{p})} C(\mathcal{D},\boldsymbol{p},\boldsymbol{p})$ is between $\frac{3}{2}$ and $2-\frac{1}{652}$.

Proof of Theorem 12 (lower bound)

- $n + 1$ points: $\{0, 1, \ldots, n\}$.
- $p_0 = \frac{1-\epsilon}{2}$, $p_i = \frac{1+\epsilon}{2n}$ for all $i > 0$.
- $d_{0,i} = 1$ for all $i > 0$, and $d_{i,j} = 1 - \epsilon$ for all $i, j > 0$.



- $c_0 = \frac{1}{2} + O(\epsilon)$, $c_i = 1 - O(1/n) - O(\epsilon)$ for $i > 0$.
- Candidate 0 loses to any other candidate in the election.
- The expected distortion is at least $(\frac{1}{2} - O(\epsilon)) \cdot (2 - O(\epsilon) - O(1/n)) + \frac{1}{2} \cdot 1 = \frac{3}{2} - O(\epsilon) - O(1/n) \rightarrow \frac{3}{2}$.

Proof of Theorem 12 (upper bound)

- Let $\delta = \frac{1}{326}$.
- Case I: all pairwise elections have distortion $\leq 3 - \delta$.
  - By Lemma 11, the overall expected distortion $C(\mathcal{D}, \boldsymbol{p}, \boldsymbol{p}) \leq (1+\alpha)/2 \leq 2 - \delta/2 = 2 - 1/652$.
- Case II: $\exists$ some pair of candidates with distortion $\geq 3 - \delta$.
  - By Lemma 13, the overall expected distortion $\leq \frac{3}{2} + 9\sqrt{\delta} \leq 2 - \frac{1}{652}$.

Lemma 13

Assume that $\delta \leq \frac{1}{100}$. Let $(\mathcal{D}, \boldsymbol{p}, \boldsymbol{p})$ be an instance with maximum pairwise distortion $3 - \delta$. Then, $C(\mathcal{D}, \boldsymbol{p}, \boldsymbol{p}) \leq \frac{3}{2} + 9\sqrt{\delta}$.

Discussion


ORACLE
B/R