A Game-Theoretic Exploration on Group Competition and Formation through Online Learning Algorithms

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– https://josephcclin.github.io

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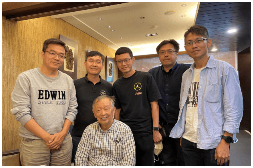
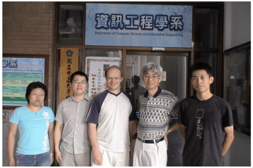
September 19, 2025

Outline

Education

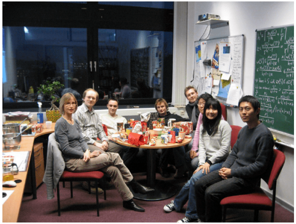
- BS.: Mathematics (2002),
  National Cheng Kung University
- MS.: CSIE (2004),
  National Chi Nan University
  - Supervisor: R. C. T. Lee
    Algorithms
- Ph.D.: CSIE (2011),
  National Chung Cheng University
  - Supervisors: Maw-Shang Chang
    & Peter Rossmanith
    FPT + Randomized Algorithms





DAAD-NSC Sandwich Program (2007–2008)

RWTH Aachen University (Funding: DAAD + NSC 96-2911-I-194-008-2.)



Postdoc in Academia Sinica (2011–2018)

研發替代役 (2011–2014)

@Genomics Research Center, Academia Sinica
- Bioinformatics,
- Comparative Genomics

PI: Trees-Juen Chuang

Academia Sinica
Genomics Research Center

@Institute of Information Science, Academia Sinica
- Machine Learning,
- Game Theory

PI: Chi-Jen Lu

Industrial Experience (2018–2020)

- *Quantitative Analyst* (intern) of Point72/Cubist Systematic Strategies (2018–2020).
    - US Hedge Fund; Fintech; Data Science.
    - Taipei Branch (started since in 2019).
    - CEO & Chairman: Steven A. Cohen.
    - AUM: US$27.2 billion (Jan. 2023).
- *Quantitative Analyst* of Seth Technologies Inc. (2020–2021).
    - High-Frequency Trading; Hedge Fund; Fintech; Data Science.
    - Taiwan based.

Cubist Systematic Strategies

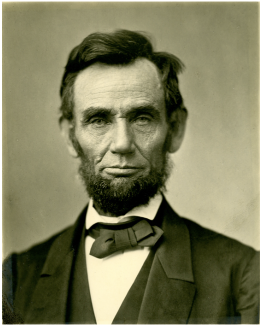
Point72 Asset Management

Photo: REUTERS/Lucy Nicholson
https://www.calcalistech.com/ctechnews/article/h1tli1t3s

Image reference: https://www.seth-tech.com/

Outline

The Inspiration



"[...] and that government of the people, by the people, for the people, shall not perish from the earth."

— Abraham Lincoln, 1863.

Motivations (I): Why The Two-Party System?

"The simple-majority single-ballot system favours the two-party system."
— Maurice Duverger, 1964.

Motivations (II): Social Choice Rules

Example:

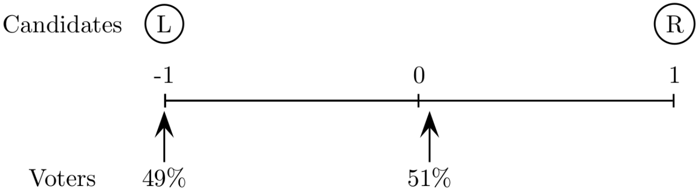
- Each voter provides an ordinal ranking of the candidates,
- Aggregate these rankings to produce either a single winner or a consensus ranking of all (or some) candidates.

Gibbard–Satterthwaite Theorem (1973)

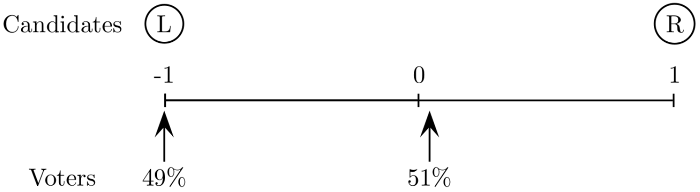
Given a deterministic electoral system that choose a single winner. For every voting rule, one of the following three things must hold:

- The rule is dictatorial.
- The rule limits the possible outcomes to two alternatives only.
- The rule is susceptible to tactical voting.

Motivations (III): Distortion of Social Choice Rules



Motivations (III): Distortion of Social Choice Rules



- The average distance from the population to candidate L: ≈ 0.5.
- The average distance from the population to candidate R: ≈ 1.5.
- But R will be elected as the winner in the election.

Issues of Previous Studies

- Voters' behavior on a micro-level.
  - Voters are strategic;
  - Voters have different preferences for the candidates.
  - Various election rules result in different winner(s).

    ⋮

Our Contribution (I)

- First, we consider an intuitive macro perspective instead.
  - Parties are players;
  - The strategies can be their nominated candidates (or policies);

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  - The point is:
    - who is more likely to win the election campaign?
    - Is the game "stable" in some sense?
    - What's the price for stability which resembles "the distortion"?

Our Contribution (II)

- Second, we simulate the behaviors of myopic strategic agents.
  - group-joining strategies
  - opinion updates
  - opinion updates with regularization
    - not willing to deviate from their beliefs too much.

Outline

Two-Party Election Game

Coauthors

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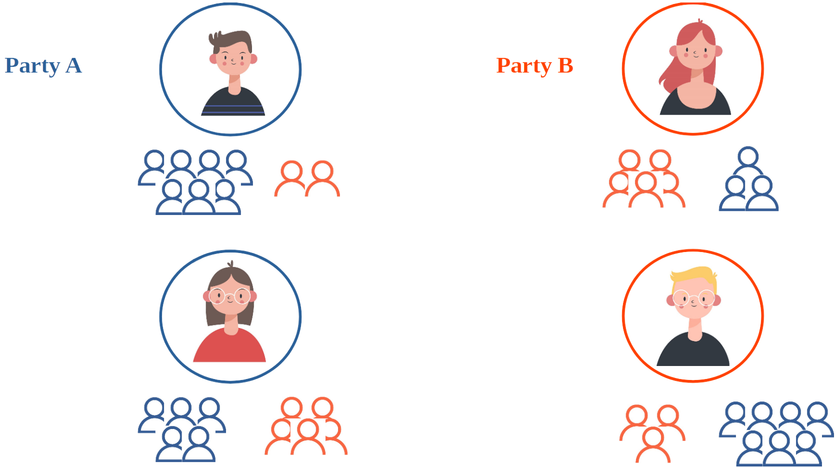
TW

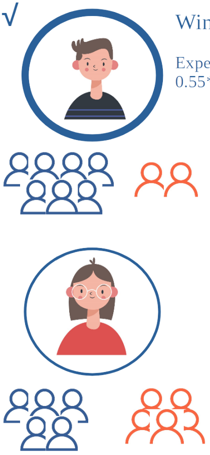
Po-An Chen
IIM, NYCU
TW


Party A
Party B

Two-Party Election Game: Formal Setting

- Party A: m candidates, party B: n candidates.
- Candidate A_i can bring social utility $u(A_i) = u_A(A_i) + u_B(A_i) \in [0, \beta]$ for some real $\beta \geq 1$.
- $p_{i,j}$: $\Pr[A_i$ wins over $B_j]$.
  - Linear link: $p_{i,j} := (1 + (u(A_i) - u(B_j))/\beta)/2$
  - Natural: $p_{i,j} := u(A_i)/(u(A_i) + u(B_j))$
  - Softmax: $p_{i,j} := e^{u(A_i)/\beta}/(e^{u(A_i)/\beta} + e^{u(B_j)/\beta})$
- Reward $r_A = p_{i,j} u_A(A_i) + (1 - p_{i,j}) u_A(B_j)$.

Party A

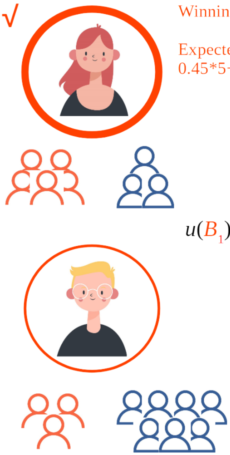


Winning prob.=0.55

Expected utility for A:
0.55*7+0.45*3 = 5.2

$u(A_1) = 7 + 2 = 9$

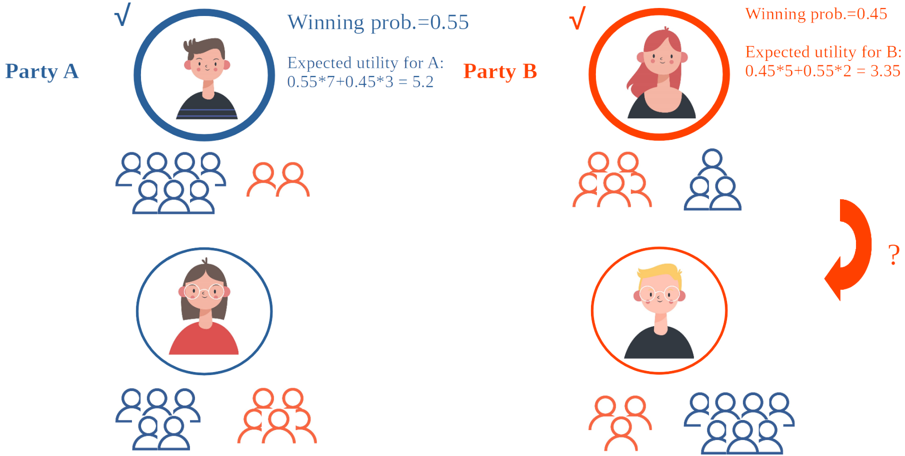
Party B

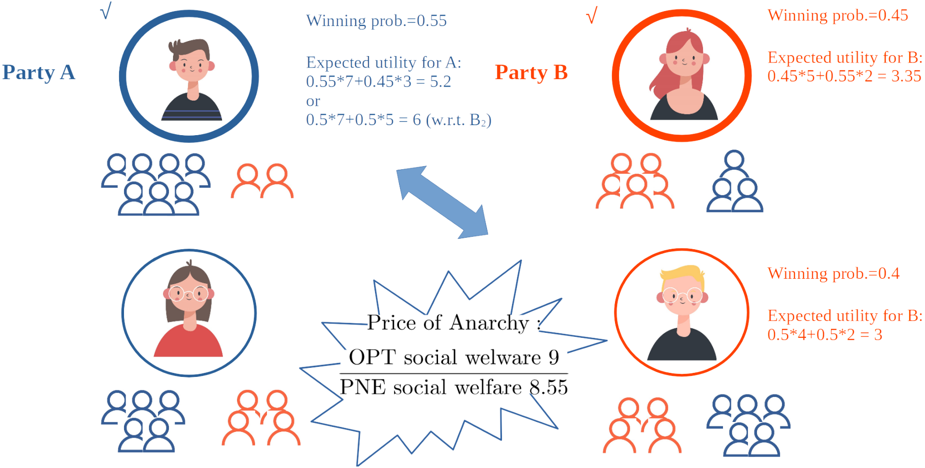


Winning prob.=0.45

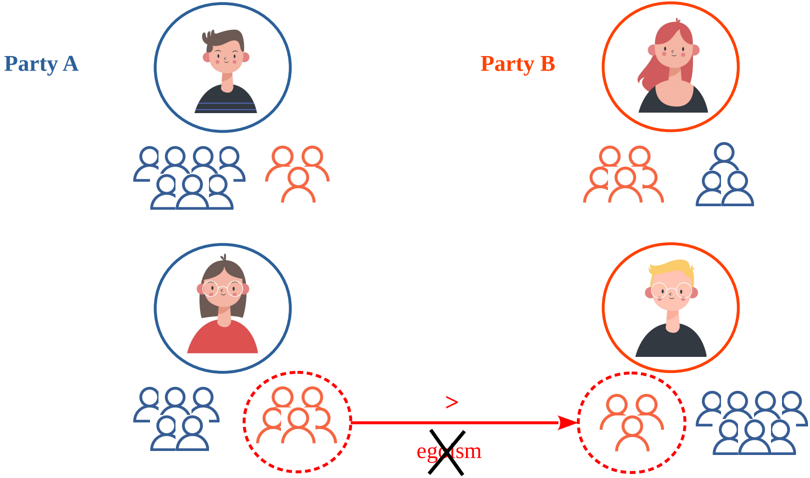
Expected utility for B:
0.45*5+0.55*2 = 3.35

$u(B_1) = 5 + 3 = 8$


√
Party A
Winning prob.=0.55
Expected utility for A:
0.55*7+0.45*3 = 5.2
Party B
√
Winning prob.=0.45
Expected utility for B:
0.45*5+0.55*2 = 3.35
?


√
Party A
Winning prob.=0.55
Expected utility for A:
0.55*7+0.45*3 = 5.2
or
0.5*7+0.5*5 = 6 (w.r.t. B₂)
√
Party B
Winning prob.=0.45
Expected utility for B:
0.45*5+0.55*2 = 3.35
Price of Anarchy :
OPT social welware 9
PNE social welfare 8.55
Winning prob.=0.4
Expected utility for B:
0.5*4+0.5*2 = 3

Egoism (Selfishness)



```
# iteration 9625:
Party A's candidates: (446, 4), (323, 372)
Party B's candidates: (503, 84), (428, 262)

     240.203,      287.682           331.92,       266.88
     216.406,      430.426          292.653,       399.86
PoA updated: 1.31186

# iteration 78207:
Party A's candidates: (530, 13), (420, 362)
Party B's candidates: (485, 58), (405, 317)

         294,          249          404.437,      244.084
     282.259,      408.802           371.59,       382.21
PoA updated: 1.38821

# iteration 1440494:
Party A's candidates: (552, 16), (517, 52)
Party B's candidates: (667, 6), (483, 508)

     250.335,      375.678          520.694,       348.27
     234.928,       391.48          510.601,      358.441
PoA updated: 1.38822

# iteration 3280308:
Party A's candidates: (361, 33), (230, 363)
Party B's candidates: (436, 7), (374, 212)

     175.327,      244.373          272.196,      236.236
     135.225,      394.025          221.063,      368.462
PoA updated: 1.40463

# iteration 5403654:
Party A's candidates: (393, 55), (332, 293)
Party B's candidates: (402, 35), (332, 312)

     215.969,      226.592          344.562,      220.646
     211.418,      337.254           321.81,       312.87
PoA updated: 1.43411
```


...
從實驗結果來看，
pure NE 一定存在對吧？
...
可以證明得出來對吧．．．

Counterexamples (Natural function)

A		B	
$u_A(A_i)$	$u_B(A_i)$	$u_B(B_j)$	$u_A(B_j)$
91	0	11	1
90	8	10	20

A		B	
$u_A(A_i)$	$u_B(A_i)$	$u_B(B_j)$	$u_A(B_j)$
44	10	37	17
39	55	10	5

	B_1	B_2
A_1	$a_{1,1}$, $b_{1,1}$	$a_{1,2}$, $b_{1,2}$
A_2	$a_{2,1}$, $b_{2,1}$	$a_{2,2}$, $b_{2,2}$

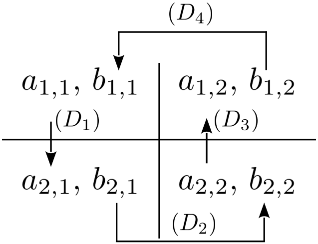
$\approx$

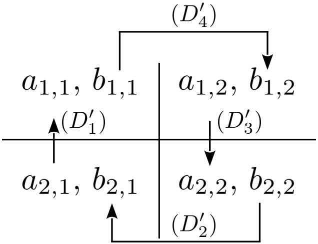
	B_1	B_2
A_1	80.51, 1.28	73.84, 2.17
A_2	80.29, 8.32	74.02, 8.23

,

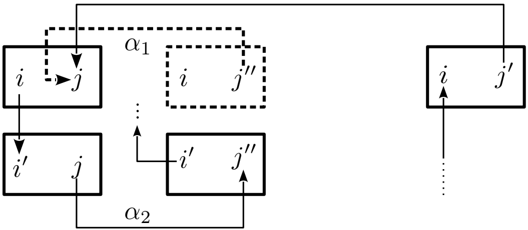
	B_1	B_2
A_1	30.50, 23.50	35.52, 10.00
A_2	30.97, 48.43	34.32, 48.81

The Deviation Cycles





Extending to general cases $m, n \geq 2$



PoA (Linear & Softmax; tight)

A		B	
$u_A(A_i)$	$u_B(A_i)$	$u_B(B_j)$	$u_A(B_j)$
ϵ	0	ϵ	0
$\epsilon - \delta$	$\epsilon - \delta$	$\epsilon - \delta$	$\epsilon - \delta$

	B_1	B_2
A_1	$a_{1,1},\ b_{1,1}$	$a_{1,2},\ b_{1,2}$
A_2	$a_{2,1},\ b_{2,1}$	$a_{2,2},\ b_{2,2}$

$\approx$

	B_1	B_2
A_1	$\frac{\epsilon}{2},\quad \frac{\epsilon}{2}$	$\epsilon - \frac{\delta}{2},\quad \frac{\epsilon}{2} - \frac{\delta}{2}$
A_2	$\frac{\epsilon}{2} - \frac{\delta}{2},\quad \epsilon - \frac{\delta}{2}$	$\epsilon - \delta,\quad \epsilon - \delta$

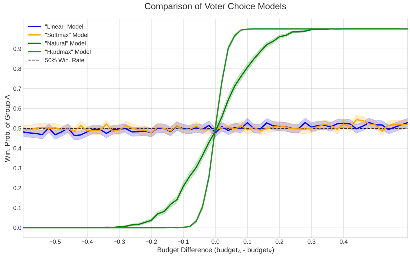
Results (Two-Party)

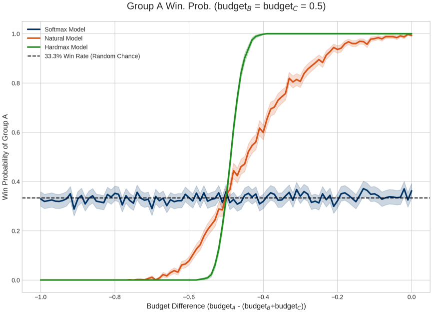
	Linear Link	Natural	Softmax
PNE w/ egoism	✓	×	✓
PNE w/o egoism	×	×	$?^{\#}$
Worst PoA w/ egoism	$\leq 2^*$	≤ 2	$\leq 1 + e$
Worst PoA w/o egoism	∞	∞	∞

- Lin, Lu, Chen: *Theoret. Comput. Sci.*, 2021.

How about the Game for Two-or-More Parties?

Verifying the Monotone Property





Example: Three-Party Election Game

- Party A, B, C: with m_1, m_2, m_3 candidates, resp.
- E.g., candidate A_i can bring social utility $u(A_i) = u_A(A_i) + u_B(A_i) + u_C(A_i) \in [0, \beta]$ for some real $\beta \geq 1$.
- $p_{i,(j,k)}$: $\Pr[A_i$ wins over B_j and $C_k]$.
  - Natural: $p^A_{i,(j,k)} := u(A_i)/(u(A_i) + u(B_j) + u(C_k))$
  - Softmax: $p^A_{i,(j,k)} := e^{u(A_i)/\beta}/(e^{u(A_i)/\beta} + e^{u(B_j)/\beta} + e^{u(C_k)/\beta})$
- Reward $r_A = p^A_{i,(j,k)} u_A(A_i) + p^B_{j,(i,k)} u_A(B_j) + p^C_{k,(i,j)} u_A(C_k)$.

k-Party Election Game, $k \geq 2$

- Party $A, B, C, \ldots$: with $m_1, m_2, m_3, \ldots$ candidates, resp.
- E.g., candidate A_i can bring social utility $u(A_i) = u_A(A_i) + u_B(A_i) + u_C(A_i) + \ldots \in [0, \beta]$ for some real $\beta \geq 1$.
- $p^A_{i,(j,k,\ldots)}$: $\Pr[A_i$ wins over the other candidates$]$.
    - Consider all monotone winning probability functions.
        - E.g., $p^A_{i,(-i)} \geq p^A_{i',(-i)}$ whenever $u(A_i) \geq u(A_{i'})$.
- Reward $r_A = p^A_{i,(j,k)} u_A(A_i) + p^B_{j,(i,k)} u_A(B_j) + p^C_{k,(i,j)} u_A(C_k) + \cdots$

PoA in General & FPT

Bad News

Three-party election games do not always have a PNE, even it is egoistic.

Theorem

For any k-party election game, $k \geq 2$, we have PoA $\leq k$ if

- The winning probability function is monotone.
- The game is egoistic.

Theorem

To compute a PNE of the egoistic k-party election game is NP-hard but FPT ($+$natural parameters).

Key Propositions

Proposition

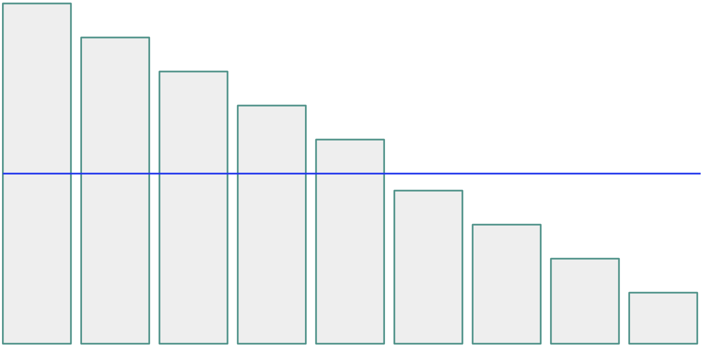
Let $\mathrm{s} = (s_i)_{i\in[m]}$ be a PNE and $\mathrm{s}^* = (s_i^*)_{i\in[m]}$ be the optimal profile. Then, $\sum_{i\in[m]} u(s_i) \geq \max_{i\in[m]} u(s_i^*)$.

Two Important Observations

$$SW(\mathrm{s}) = \sum_{1\leq i\leq k} p_{i,\mathrm{s}} \cdot u(s_i) \leq \max_{1\leq i\leq k} u(s_i)$$

$$SW(\mathrm{s}) = \sum_{1\leq i\leq k} p_{i,\mathrm{s}} \cdot u(s_i) \geq \frac{1}{m} \cdot \sum_{1\leq i\leq k} u(s_i).$$

An idea on the train



Outline

Coauthors

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Group Formation

- n agents $v_1, v_2, \ldots, v_n$ with opinions $\mathbf{z} = (z_1, z_2, \ldots, z_n)$ and beliefs $\mathbf{s} = (s_1, s_2, \ldots, s_n)$.
- Agents in groups $\mathcal{G} = (G_1, G_2, \ldots, G_m)$.
- State of the game: $\tau = (\mathbf{z}, \mathbf{s}, \mathcal{G})$
- Winning prob. of group G_j:

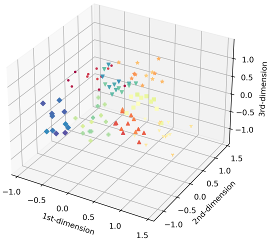
$$p_j(\tau) = \frac{e^{n_j \langle \bar{g}_j, \sum_{v_r \in V} s_r \rangle}}{\sum_{i \in [m]; n_i > 0} e^{n_i \langle \bar{g}_i, \sum_{v_r \in V} s_r \rangle}},$$

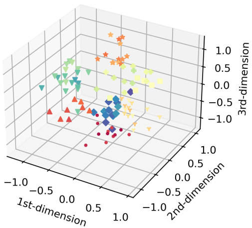
- Reward of agent i:

$$r_i(\tau) = \sum_{j=1}^{m} p_j(\tau) \langle s_i, \bar{g}_j \rangle,$$

Group Formation

- By the group joining strategy:
  - $j = \arg\max_{\ell} p_{\ell}(\tau) \cdot \langle \bar{g}_{\ell}, s_i \rangle$, where $\bar{g}_j = \sum_{v_i \in G_j} z_i / |n_j|$: the opinion of group G_j.

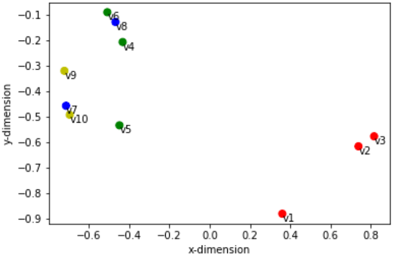


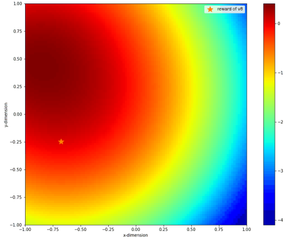


Group Formation

- Multiagent Online Gradient Ascent + Regularization

$$r_i(\tau_t) = \sum_{i=1}^{m} p_j(\tau_t)\langle s_i, \bar{g}_j \rangle - \|z_i - s_i\|_2^2.$$





- τ_t: state at time t; p_j: win. prob. of group j;
- z_i, s_i: opinion and belief of agent i respectively; $\bar{g}_j$: avg. opinion of group j.

Group Formation (IEEE CIM - AI-eXplained)

Group Formation by Group Joining and Opinion Updates via Multi-Agent Online Gradient Ascent

An interactive article on illustrating group joinging strategies and opinion updates via online gradient ascent to analyze group formation dynamics. Learn how the choices of coaltion of agents lead to a pure-strategy Nash equilibrium and how updating agents' opinions eventually stablizes group formation.

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Published
XXX. X, 2023

Indicates interactive elements

Contents:

Thoughts

- Asking good questions is important.

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- Keep moving forward after a series of rejections.

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Thoughts

- Asking good questions is important.
- Keep moving forward after a series of rejections.
- Keep learning and enjoy the process.
- To teach to learn.

Mottos

“Think hard, and work smartly.” – R. C. T. Lee & Maw-Shang Chang

“Every job is a self-portrait of the person who did it. Autograph your work with quality.” - Prof. D. T. Lee

Thank You.