How Bad Can an Election Game of Two or More Parties Be?

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a joint work with

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6 May 2024

Lin, Lu, Chen

Election Game @ GAIW'24

6 May 2024

Authors



Chuang-Chieh Lin



Chi-Jen Lu



Po-An Chen

Outline







Motivations

Outline



Our Contribution



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Election Game @ GAIW'24

Motivations

The Inspiration (an EC'17 paper)



"[...] and that government of the people, by the people, for the people, shall not perish from the earth."

— Abraham Lincoln, 1863.

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Motivations

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Most Previous Studies from a Micro Perspective

- Strategic behaviors of voters.
- Design of ballots.
- Social choice function or voting rules.

The "Macro" Setting

- Instead, we consider an intuitive macro perspective instead.
 - Parties are players.
 - Strategies: their candidates (or policies).

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 - Parties are players.
 - Strategies: their candidates (or policies).
 - A candidate beats the other candidates from other candidates of other parties with <u>uncertainty</u>.
 - The payoff of each party: expected utility its supporters can get.
 - The egoistic property: each candidate of party \mathcal{P} brings more utility to \mathcal{P} 's supporters than any candidate from the other parties does to \mathcal{P} 's supporters.

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Two-Party Election Game: Formal Setting

- Party A: m candidates, party B: n candidates.
- Candidate A_i can bring social utility u(A_i) = u_A(A_i) + u_B(A_i) ∈ [0, β] for some real β ≥ 0.
- $p_{i,j}$: $Pr[A_i \text{ wins over } B_j]$.
 - Linear: $p_{i,j} := (1 + (u(A_i) u(B_j))/\beta)/2$
 - Natural: $p_{i,j} := u(A_i)/(u(A_i) + u(B_j))$
 - Softmax: $p_{i,j} := e^{u(A_i)/\beta} / (e^{u(A_i)/\beta} + e^{u(B_j)/\beta})$

• Payoff (reward) $r_A = p_{i,j}u_A(A_i) + (1 - p_{i,j})u_A(B_j)$.

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- $p_{i,i}$: $\Pr[A_i \text{ wins over } B_i]$. more utility for all the people, more likely to win
 - Linear: $p_{i,i} := (1 + (u(A_i) u(B_i))/\beta)/2$
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• Payoff (reward) $r_A = p_{i,i} u_A(A_i) + (1 - p_{i,i}) u_A(B_i)$.

Motivations

Price of Anarchy (poA)



Election Game @ GAIW'24

6 May 2024

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Election Game @ GAIW'24

Motivations

Egoism (Selfishness)

Party A



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m-Party Election Game, m > 2

- Party $\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_3, \ldots$: with n_1, n_2, n_3, \ldots candidates, resp.
- E.g., candidate s_i of party \mathcal{P}_i can bring social utility $u(s_i) = u_1(s_i) + u_2(s_i) + \ldots + u_m(s_i) \in [0, \beta]$ for some $\beta > 0$.
- $p_{i,s}$: $\Pr[s_i \text{ wins the campaign w.r.t. } s]$.
 - s: the strategy profile of all party players.
 - Consider all monotone winning probability functions.
 - E.g., $p_{i,s_{-i}} \ge p_{i',s_{-i}}$ whenever $u(s_i) \ge u(s_{i'})$.

• Payoff (reward) $r_i = p_{1,\mathbf{s}_{-i}}u_i(s_1) + p_{2,\mathbf{s}_{-i}}u_i(s_2) + \cdots + p_{m,\mathbf{s}_{-i}}u_i(s_m)$.

Outline







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- Our focus:
 - The probability for a party to win the election campaign.
 - Monotone function (more utility for all the people, more likely to win).

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- What's the price of anarchy (PoA)?
 - Upper bound: number of parties.
 - The bound is tight for some cases.

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 - Upper bound: number of parties.
 - The bound is tight for some cases.
- Incentives of forming a coalition for each party.

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Counterexamples (Natural function)

Α		В		Α		В	
$u_A(A_i)$	$u_B(A_i)$	$u_B(B_j)$	$u_A(B_j)$	$u_A(A_i)$	$u_B(A_i)$	$u_B(B_j)$	$u_A(B_j)$
91	0	11	1	44	10	37	17
90	8	10	20	39	55	10	5

	B_1	B_2	
A_1	<i>a</i> _{1,1} , <i>b</i> _{1,1}	<i>a</i> _{1,2} , <i>b</i> _{1,2}	≈
A_2	$a_{2,1}, b_{2,1}$	a _{2,2} , b _{2,2}	-

	B_1	B ₂			B_1	B ₂
A_1	80.51, 1.28	73.84, 2.17	-,	A_1	30.50, 23.50	35.52, 10.00
A_2	80.29, 8.32	74.02, 8.23	-	A_2	30.97, 48.43	34.32, 48.81

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Counterexamples (Softmax; Three parties)

$u_1(x_{1,i})$	$u_2(x_{1,i})$	$u_3(x_{1,i})$	$u_1(x_{2,i})$	u ₂ (2	$x_{2,i}) u_3(x)$	$u_{2,i}) u_1(x)$	$u_{3,i}) u_2(x_{3,i})$	$_{i}) u_{3}(x_{3,i})$
29	4	21	23	59	7	8	32	54
27	43	3	3	57	38	20	13	53
	r ₁ (1 - 1 - 1)	Ko (a a a	Ka (a a	.	K ₁ (1 1 0)	K ₀ (1 1 0)	r ₀ (1 + 0)	
	1,(1,1,1)	12,(1,1,1) '3,(1,1	,1)	1,(1,1,2)	12,(1,1,2)	73,(1,1,2)	\sim
	$r_{1,(1,2,1)}$	$r_{2,(1,2,1)}$) $r_{3,(1,2)}$,1)	$r_{1,(1,2,2)}$	$r_{2,(1,2,2)}$	$r_{3,(1,2,2)}$	
	18.81	34.64	28.5	51	23.49	27.82	2 27.3	8
	11.27	34.67	39.7	'0	15.57	28.09	38.9	3
	$r_{1,(2,1,1)}$	$r_{2,(2,1,1)}$) $r_{3,(2,1)}$,1)	$r_{1,(2,1,2)}$	$r_{2,(2,1,2)}$	<i>r</i> _{3,(2,1,2)}	- ~
	$r_{1,(2,2,1)}$	$r_{2,(2,2,1)}$) $r_{3,(2,2)}$,1)	$r_{1,(2,2,2)}$	$r_{2,(2,2,2)}$	<i>r</i> _{3,(2,2,2)}	
	18.74	44.53	22.8	34	23.18	38.35	5 21.6	1
	11.58	44.25	33.6	6	15.67	38.27	7 32.7	7

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Previous Results (Two-Party)

	Linear	Natural	Softmax
PNE w/ egoism	\checkmark	×	\checkmark
PNE w/o egoism	×	×	?#
Worst PoA w/ egoism	$\leq 2^*$	<u>≤</u> 2	$\leq 1 + e$
Worst PoA w/o egoism	∞	∞	∞

• Lin, Lu, Chen: Theoret. Comput. Sci., 2021.

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Complexity & PoA Bounds for m > 2 Parties (GAIW'2024)

Non-Existence of a Pure-Strategy Nash Equilibrium

The three-party election game does NOT always have a PSNE.

Theorem

For any *m*-party election game, m > 2, we have PoA < m if

- The winning probability function is **monotone**.
- The game is egoistic.

Theorem

To determine if a PSNE exists in the egoistic *m*-party election game is NP-complete but FPT (+natural parameters).

- The game instance is in a succinct representation.
- A reduction from the SAT problem.

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Election Game @ GAIW'24

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Key Propositions

For the egoistic election game:

Proposition

Let
$$\mathbf{s} = (s_i)_{i \in [m]}$$
 be a PSNE and $\mathbf{s}^* = (s_i^*)_{i \in [m]}$ be the optimal profile.
Then, $\sum_{i \in [m]} u(s_i) \ge \max_{i \in [m]} u(s_i^*)$.

Two Important Observations

$$\begin{split} \mathcal{SW}(\mathbf{s}) &= \sum_{i \in [m]} p_{i,\mathbf{s}} \cdot u(s_i) \leq \max_{i \in [m]} u(s_i) \ \mathcal{SW}(\mathbf{s}) &= \sum_{i \in [m]} p_{i,\mathbf{s}} \cdot u(s_i) \geq rac{1}{m} \cdot \sum_{i \in [m]} u(s_i). \end{split}$$

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Shrinking nominating depth of a party



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6 May 2024

Conclusion

Outline

Motivations

2 Our Contribution



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Concluding Remarks

- We assume the utility is evenly distributed to the voters.
- The PoA is small in most game instances (simulations).
- We will conduct experiments to simulate voters' voting decisions to see how *monotone* the winning probability function is.

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Conclusion

Thanks for your attention!

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